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ABSTRACT

As part of a 3-year comprehensive interdisciplinary program developed by a group of educators from Hackensack High School, New Jersey, this teaching guide for a Grade 10 mathematics unit is designed as a year long study of measurement in preparation for further technical study in Grades 11 and 12. Daily lesson plans for the four sophomore units stress basic concepts and applications of mathematical measurement. Students construct models of ductwork, geometric solids, densities of metals, and paper box fabrication which promote group and individual participation in developing necessary concrete mathematical skills. The program incorporates the use of community resources for field trips and presentations, and includes line diagrams, quizzes, student worksheets, and activity lists. Introductory rationales precede each outlined unit. This volume is planned for use with four others, available as VT 015 228-VT 015 231 in this issue. (AG)

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INDUSTRIAL PREP

Volume One

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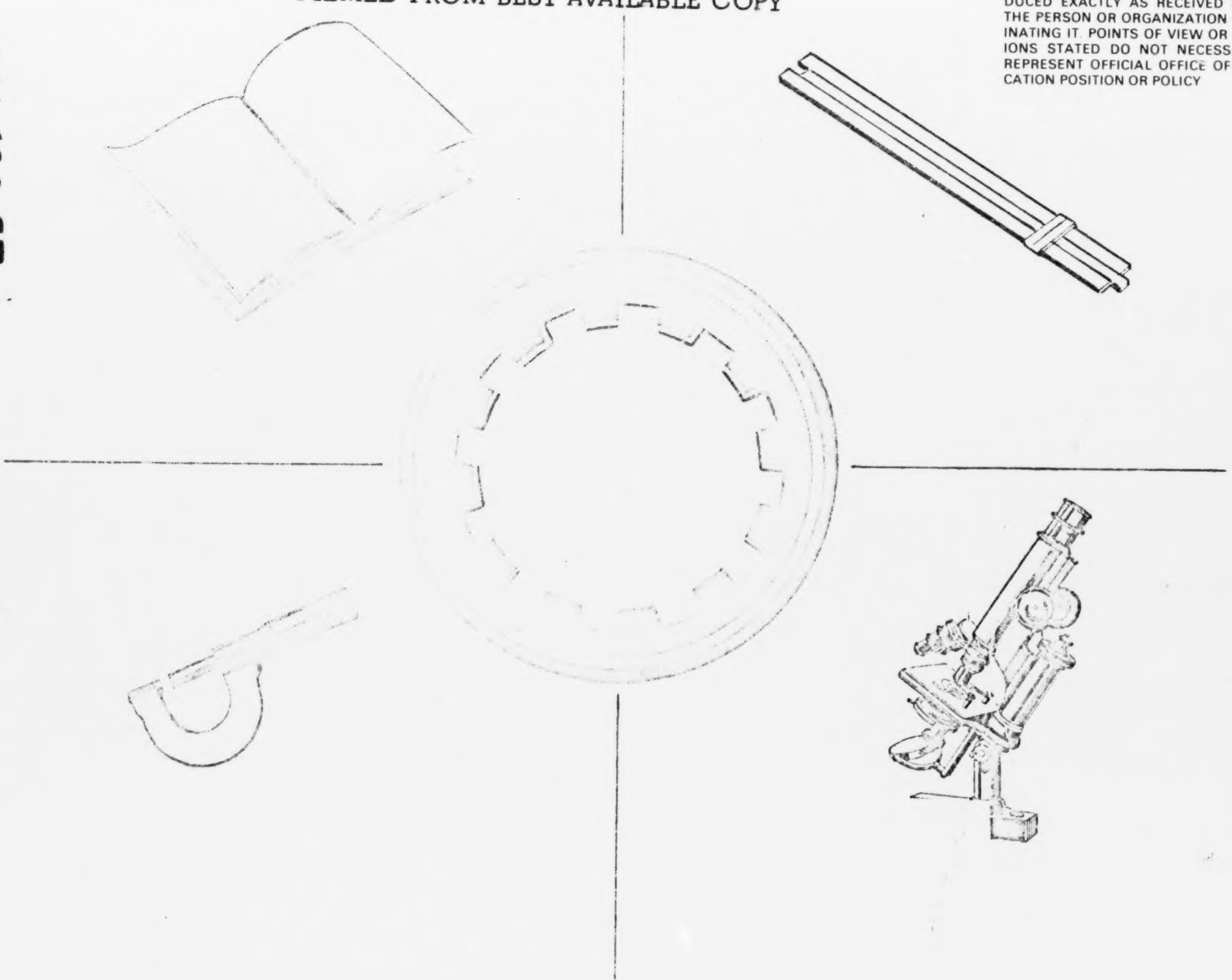

Sophomore Year

INTRODUCTION

MATHEMATICS

Hackensack High School

VT015227

E-8-1

9-12 Explanation of plan

H00532

WHAT ARE THE STUDENT ACTIVITIES ASSOCIATED WITH THIS UNIT: (discuss the amount of time as well as the characteristics of the activities)

A wide variety.

Is there a written description of these activities? yes

WHAT ARE THE TEACHER ACTIVITIES ASSOCIATED WITH CONDUCTING THIS UNIT:

See Materials.

Is there a written description or manual documenting these activities? yes.

WHAT PREPARATION IS REQUIRED OF A TEACHER PRIOR TO UNIT INSTALLATION:

Orientation, plus the manual.

IS there a written manual of description of the preparation which could be used by a new teacher using this unit? ~~yes~~

HOW MUCH CLASS TIME IS REQUIRED FOR THIS UNIT: (e.g. hours per week. Be sure to indicate if out-of-school time is required of the students)

IS THE UNIT BEING USED IN OR DOES IT RELATE TO ANY OF THE FOLLOWING SUBJECT AREAS:

x English
x Math
x Science
x Voc. Ed.
___ Social Studies
___ Foreign Lang.
___ Art
___ Music
___ Phys. Ed.
___ Health
___ Other (specify)

ARE THERE ANY SPECIAL PROBLEMS, LIMITATIONS, OR, GOOD POINTS THAT YOU FEEL SHOULD BE KNOWN ABOUT THIS UNIT:

12/3/71 816

CODE # : __________

School(s): HIGH SCHOOL

Grade(s) : 9 - 12

Curr. Unit Inventory

HACKENSACK

TITLE: Industrial Prep

GENERAL PURPOSE: To prepare students to meet all of their societal rolls including work.

SPECIFIC GOALS:

The student will apply his total education to his daily living.
The student will understand himself.
The student will be a multi-faceted, flexible well-educated person.
The student will understand that cooperation is essentially more important an attribute than competitiveness.
The student will understand that the total community is his education.

PERFORMANCE OBJECTIVES:

The student will understand that the interdependence of disciplines should be an essential appreciation as well as being a truly visable means of good teaching.

Specific performance objectives have not been prepare for this unit.

WHAT UNIQUE MATERIALS ARE REQUIRED FOR USE OF THIS UNIT:

The unit as prepared.

WHAT IS THE SOURCE OF THESE MATERIALS: (e.g. teacher constructed, purchased from a publisher, donated from somewhere, etc.)

Teacher constructed.

WHERE CAN THESE MATERIALS BE SEEN OR OBSERVED: Career Education Center
Hackensack

HOW MANY STUDENTS ARE INVOLVED IN THIS UNIT: 70
HOW MANY TEACHERS ARE INVOLVED WITH THIS UNIT: 6
HOW MANY COUNSELORS ARE INVOLVED WITH THIS UNIT: 2

IS THIS A NEW UNIT OR HAS IT BEEN USED BEFORE: (if it is not new then how many years/months etc. has it already been used and in how many classrooms)

Used for six year.

Introduction

Industrial Prep is a prevocational, interdisciplinary program. It was developed in Hackensack High School because of the need to provide a curriculum that would be consistent with the demands placed people entering occupations in the 1970's.

Educaters have jumped from one extreme to another during the last two decades. In the 1950's the magic word was "gifted", and now we are becoming fully mobilized to meet the problems brought on by the "disadvantaged." Along the way we have neglected to stop and consider the majority of the population the so-called 'average' people, who are to become the backbone of our nation's work and life forces. This program takes these people into mind as well as the others.

We believe that the development of manhood is more important than the development of manpower. We also feel that our obligation as educators is to help people prepare to meet all of their societal roles, including work. That means that we should be able to help young people get ready for a personally relevant vocational future. However, at the same time we expend much effort in seeing that this program includes materials and experiences that work toward the total development of the individual.

When consideration and planning for the Industrial Prep Program began, a few guidelines were established. We based the program on the following organizational assumptions:

1. That we would receive no money to help us for either salaries, materials, or equipment.
2. That our then existing facilities would have to be utilized with no hope for modification or addition to them.
3. That teachers for the program would be recruited from our present staff.

Essentially we felt that no windfall would find its way to us and that we would have to use what we had, but in a different way.

The first year of preparation was devoted to the researching and the gathering of insights so as to develop a relevant, logical philosophy. Besides the reading of books, journals, and periodicals of all types, we spent a good deal of time in the field. The field being many of the major and smaller business and industrial concerns in the metropolitan area. Frequently individual and collective groups of employers and employees were invited to the school for discussion. The talk centered primarily on asking these people, "what are the basic characteristics of a promising employee?' The responses gathered from these interviews along with the materials read in the research were to become the foundations of our program.

Some of the tenets that we adopted because of this preliminary work are:

1. That a person should be able to apply his total education to his daily living and in order for him to do so he must be taught well, with useful materials.
2. That as Donald Super states, "self-knowledge is prerequisite to self-determination." Before either vocational or social decisions are to be carefully made a person must understand himself.
3. That the technical world calls for a multi-faceted, flexible well educated person.
4. That cooperation is essentially more important an attribute than competitiveness.
5. That the total community is the educator of a student, not just the school.
6. That the interdependence of disciplines should be an essential appreciation by each student as well as being a truly visable means of presenting teaching material by the staff.

Taken one at a time, these are not earth shaking contemporary thoughts, but, absorbed into a prevocational secondary setting, they become a unique set of premises on which to found a program.

In meetings with the people who work in and hire for industries, we did not come across any who held that special skills taught in high school were absolutely necessary for employment. Nobody told us that a trade learned in school was the passport to instant industrial success. In line with this is an article that appeared in the April 4, 1965 issue of the New York Times. It told of a minority report of an extensive study by an eleven man vocational education commission in Nassau County, New York. A section of this study "pleaded for recognition that fewer occupations than is generally believed, accept specific school-acquired skills as a prerequisite for employment." This 44 page report was never published due to conservative opposition. What we found requested by employers was a need for high school graduates who essentially could read, write, and be able to apply mathematics and scientific fundamentals to work problems.

In developing the program, we bore Dr. Super's consideration in mind. In the available literature that we were exposed to there was nothing that corroborated the wisdom and stability of an early career choice. In fact, everything cautioned against this. Therefore the Industrial Prep Program offers ample room for the exploration of vocations along with provision for self-understanding necessary to make such a decision.

So much has been said for the necessity for man to be educated for change that to elaborate on this would be presuptvous. However, I think that Robert Hutchins, former President of the University of Chicago and the present Director of The Center For Democratic Institutions advances an observation that we believe in. 'The most obvious fact about society is that the more technological it is the more rapidly it will change. It follows that in an advanced technological society futility dogs the footsteps of those who try to prepare the child for any precise set of conditions. Hence the most impractical education is the one that looks most practical, and the one that is most practical in fact is the one that is commonly regarded as remote from reality, one dedicated to the comprehension of theory and principles. In the present state of technology, and even more certainly in any future state thereof, the kind of training and information that is central in American education is obsolescent, if not obsolete. Now, the only possible adjustment that we can give the child is that which arises through putting him in complete possession of all his powers.' Our aim is to provide as comprehensive as possible an exposure to life and the tools of living and working so that the Industrial Prep student will be equipped for change.

A quote from the Kaiser Aluminum News of November 1, 1968 illustrates the point to be made in promoting cooperation. 'As the grade system has traditionally been used in the past, each student is pitted against the other. Yet in the 'real' world in which he will live as an adult, his most important ability will be his willingness and skill in working cooperatively with others. This is particularly true in the business world, which is predominantly a cooperative enterprise and not a competitive one. No automobile ever got designed, engineered, produced and distributed without the cooperation of literally thousands of people. Competition occurs only in the ultimate market place.

The Industrial Prep Program recognizes the limits and fallacy of the school as being the sole educator of a student and at the same time appreciates the potential of the community at large to take part in education. Tapping community resources in an integral feature in presenting many parts of the program.

Finally, in reviewing the program foundations, we come to the heart of the means of implementation; that being an interdisciplinary approach to education. By correlating the efforts between core areas of the program we feel that we are better able to bring significant meaning, interest, and enjoyment to learning. In retrospect, this has for the most part been borne out. To identify the natural relationships between disciplines and use them to enhance a learning situation is the key to the plan.

What we in Industrial Prep are trying to do is to present what Robert Hutchins maintains is a liberal education but with a flair toward the occupational. He says, "eliminate neither training nor the imparting of information, but use them in a different fashion." This we try to do.

The approach to the interdisciplinary scheme is similar to the Richmond Plan, but includes a different series of teaching units for different people. The interdisciplinary team that we use is made up of people from the mathematics, science, English, and industrial arts departments. Resource people from within the school that are integral parts of the program come from the social science, guidance, and special services areas. Our use of resource people from without the school includes men and women from numerous specialities and fields.

The method of correlation focuses on central problems. Each year of the three year program contains units that have a commonality of basic properties that are relevant to the participating teaching units. For instance, in the first year (the sophomore year) the basic theme is measurement and the guiding subject is mathematics. In each of the years there is a different guiding subject, one which sets the pace of correlation by the depth and amount of work covered in that class. Correlation in not done on a daily basis, nor ever forced. If a natural relationship exists between instructional areas in particular units it is capitalized on to reinforce learning and to make it commonly relevant to the total learning going on in the program.

For the measurement theme there are four projects that are used to explicity bring the teaching areas together. They include such disparate names as duct work, geometric patterns, properties of metals, and packaging.

The first project of duct work is an effective opener for us. It works out to be a scaled down air conditioning duct system made out of a few materials. We like it because it does a good job of introducing the interdisciplinary approach to the students, it's good for the development of mathematical fundamentals, it enables the youngsters to achieve positive tangible results from their theoretical learning, it introduces the concept of cooperative work, and it permits the development of an occupational plan. This is how it works.

The mathematics instructor brings the boys to a point where they can lay out a duct section using basic arithmetic. He has them fabricate this with cardboard in his classroom, giving them an opportunity to engage in manipulative work is an academic setting. This is done on a group effort with students of varying abilities intermingled. What happens in that simpler pieces of the system are made by slower students and more complicated parts, such as a transition piece, is constructed by a more able person. The boys in the group must have their pieces fit together and this affects characteristics of individual responsibility as well as all-out cooperation.

In the drafting room the unit is simply drawn giving the students the opportunity to become attentive to precision as well as introducing basic drafting techniques.

Besides developing a related technical vocabulary with the boys, which is a common enough approach to interdisciplinary work and certainly not an exciting part of it, the English teacher creates an inter-personal work atmosphere by utilizing a Tele-Trainer borrowed from the Bell System. This device is used to role-play a problem condition set-up between a customer and an employee of a heating and ventilating company. Students prepare and act-out a situation that might sound like this: 1. the customer calls to complain about lact of heat in a house, 2. complaint is accepted by employee with tact and understanding, 3. employee tries to troubelshoot over the phone, i.e., did you check emergency switch?, 4. employee then evaluates and acts on disposition of compaint. All of this is dramatized with much side-play of conversation and is recorded and played back for student analysis as to not only diplomacy, effect of communication, but also for speech and style of delivery.

Biology is a school required subject in the sophomore year. It lends itself to this unit by providing the students with an exploratory series of experiences with the human circulatory system. Nothing in depth, but just an overview is offered. Along with this, a tree's duct system is also discussed.

The metal shop is reserved for these students so that they will have some opportunity to fabricate the group duct systems in a shop. All of the boys do not take metal shop at one time, they may elect other industrial arts areas so other arrangements must be made. For instance, the metal and drafting classes might exchange periods and thus give the Industrial Prep students a chance to occupy the shop together. All industrial arts instructors, that can be spared at that time, join forces to give as much concentrated assistance as possible for the project.

A resource person from the social science department presents a program on the ways people heat and cool their buildings around the world. This is done with a profusion of visual aids and delivered in a relaxed atmosphere as a general interest program.

An occupations unit on sheet metal, air conditioning, and heating trades is correlated to the mojor project. Representatives from occupations in these fields are invited in to be interviewed by the boys. No speeches are given but rather the students ask objective and subjective questions of the visitors so as to obtain a comprehensive background about each of the job areas.

To further make the total project more relevant, short, period long field trips are taken to local business and shops that engage in related tasks to the units covered.

This type of correlation is not forced nor scheduled so as to prove inconvenient to the participating teachers. If an instructor in a particular discipline feels that he needs more time to develop some teaching material and that this might hamper the correlation schedule, then at the weekly meetings other arrangement are made. What we have found is that there have been very few occasions where total involvement was not possible. The extent of the interrelation of subjects in any unit is dependent upon the imagination and creativity of the team.

At weekly meetings the instructors themselves receive a broadened education because of the necessity of each of them knowing what is going on in the other guy's class. In order for a person to present a correlated unit he must have an idea of not only what is scheduled in the other classes but should have a working understanding of the instructional matter.

To further demonstrate the methods used in relating all of the areas we can summarize the next sophomore year unit; that of geometric patterns. In this section the mathematics teacher develops, what the teachers feel is the foundations for building the academic proficiencies and methods of attach for further learning. Here the math man combines the abstract with the cognitive in having the boys sharpen their arithmetic tools. The English instructor, with the assistance of both the drafting and mathematic's people takes the students on a world tour, using slides and narration to see the designs and patterns of nature and those developed by man in interesting settings. At the same time the free reading library in his classroom features books and magazines that reinforce this unit, while a field trip to the Whitney Museum in New York is arranged with a guide to show the boys the geometric designs in art.

Occupationally, the field of architecture fits very well into this unit. Inter-class correlation is easily woven into a unity like this because the technical nature of the material plus the abundant availability of similar subject matter in various settings blend well together.

The junior year of the program features physics as the guiding subject. This is an applied physics class. More work time is spent in the laboratory than in a lecture room. The instructor of this class has developed a detailed program guide for the applied physics work that is especially geared to capitalize on the mathematical foundations acquired by the boys during the sophomore year as well as their increased abilities in problem solving situations. The physics class is a practical, exploratory experience for Industrial Prep students. Many of the projects worked on in the labs have been designed by our instructor. His methods of teaching mechanical advantage and other areas of physics are most unique. He uses everything from surf casting rods to bottle openers to get the youngsters to discover the basic, working theories in physics.

We tie in second year mathematics very closely with physics and also include directly related English units on the senses, critical thinking, how to describe and define, and science fiction.

In the senior year, chemistry is the key subject. Again this is an applied lab science with very relevant units on foods and their additives and applied, everyday chemical exploratory experiences.

We have found that because of applying theory to practical, relevant experiences, both in physics and chemistry, that the students come away from these classes with sound, fundamental science backgrounds. The chairman of the department takes a deep sense of pride in the Industrial Prep science students and often speaks of them as being better equipped than an average college prep person in truly understanding science. Our students are not rewarded for memorization, but receive their satisfactions in learning how to apply their knowledge.

Many of the Industrial Prep students enroll in a cooperative work program in their senior year. These youngsters receive an opportunity to engage in on-the-job training experiences in an occupational area of their choice. The cooperative program provides them with one half day in school and one half on the job. Beside the specific work training function of the program it also enables students the chance to become part of an adult occupational environment and to try out their various strengths and characteristics in a new social situation.

I think that a glimpse into the teaching materials used by some of our teachers would offer more insights into what makes this program a little different.

An interesting paragraph heading of the introduction for our second year English guide state that "The Automobile and Television Set Probably Teach the Student More Than the School Teacher," Based on the observations and experience that our instructors have had with the boys in the program they realized that an entirely different set of educational experience were going to have to be steadily developed so as to capitalize on every changing student interests. Working toward student interests does not preclude teachers helping boys read, write, speak, listen, and think with as much discernment and sensitivity as possible.

A unit approach that is developed for the junior year includes: work preparation television, physics, economics, and prejudice.

A feature of the first unit (work preparation) is a boy spending school time outside of the building with a representative of an occupation of his choice. He almost literally becomes a shadow to the person for a working day and in so doing gets to feel what the job is all about.

Reading materials for our English classes include such items as Consumer Reports, Motor Trend newspapers, and contemporary novels. We have found the magazine's tests and surveys are of considerable interest to youngsters and stimulate good reading habits as well as develop critical think patterns.

The work of the Langston Hughes is used, among other sources, as material for the unit on prejudice. He is simple enough to read and yet the boys can be touched by the sad, bitter-sweet humor of his writing and can be introduced to more such work by this material.

A follow-up on this unit is offered in the senior year by a series of small group sessions on occupational relations. These sessions were developed by an English and foreign language teacher to deal with possible sensitive inter-personal situations a young employee might face in a work environment. The two teachers spent a summer collecting hard-to-find factual materials about ethnic and racial minorities and came up with a series of guides to the presentation of modified T-sessions. A major part of the teachers' research and planning time was spent with people affected by such confrontation situations.

The senior year's English program has units based on work entrance, the film, chemistry, was and peace, and leisure time activities. Basing a philosophy on the supposition that the image is more significant to the student raised in the electronic age, than the printed word, the instructor brings into the program a study of film after the junior year's work in television. For this school year the class produced a short film study on the pollution of our environment. What is interesting about this is that these are supposedly limited, non-college bound youngsters who are considered to have such limited ability.

The war and peace section capitalized on a maturing boy's broader interests. The titles of parts of this unit are: The Many Faces of War, Ideas From the Great Books, Film Shorts, Full-Length Films, Short Stories and Records, Novels, and Poetry. Sounds very academic, but what the teacher does is to explore with the boys insights into why people fight, through a simple survey of the preceding media. If this plus the directly related material is presented, as Bruner says, on a level that speaks the language to the individual it can be handled and understood.

Should you wonder about a unit on leisure time as being relevant subject for a pre-vocational program I would like to quote the following from the February/March 1969 issue of Steel Facts. "During the next five years, each employee having continous service on January 1, 1969, in the lower half of his company's seniority list will, for the first time, be eligible for one extended vacation consisting of his regular vacation plus three extra weeks. Employees in the top half of their companies' seniority lists will, as they have been in the past, be eligible for one extended vacation of 13 weeks, including their regular vacations, during the five-year period."

The University of Redlands in California did a study on the senior group of steelworkers who were on such an extended vacation. They found that for the most part the men did not travel, did not engage in any civic activities, nor read or go to the theatre. What they found was that most of the time was spent in sitting in the yard drinking beer. This is not an entire waste of time, but for thirteen weeks plus about three or four for the normal vacation (which is something like four months) this can be a grossly unproductive period of a person's life, both as a contributor and as an ever developing individual. It seems as if schools have to do something about education for leisure time. The unit developed by our men has sections that include community involvement as well as introductory experiences for individual enjoyment.

There is a sophomore year unit on simple psychology that combines the efforts of the English, shop, and biology teachers as well as that of one of the school psychologists. Students build mazes, buy mice, run them through under test conditions and then engage in informal, exploratory discussions on behavior conditioning. Nothing elaborate, but the unit is presented simply to the youngsters and it is very well received. We have found that they are quite interested in human behavior and have a thrist for knowledge in this area.

A problem that we are starting to face is the fact that many of the students have had a reawakening of academic stimulation because of the way that they have been treated in the program. They have been provided with opportunities to succeed in the same areas that they had previsously been less than successful in. Some have expressed interest in college and must, although they can compete with many college prep students on applying the knowledge that they have acquired in high school, pay a penalty because their classes had names that connoted industrial or practical work. However, with the opening of a community college in our county this problem now can be dealt with on a positive basis.

The program is a small one, even though it has been said that it was designed for a large population. There are a number of reasons why it is small. Like most schools in the metropolitan area we have parents and children who think that there is nothing really worthwhile except a college education. We are still battling that concept. I think that we are now in a position to grow because word of mouth has spread and we find many more people interested all of the time. Another reason that we are small in number is that because of the nature of the interdisciplinary approach there is a dual responsibility on teachers. They still belong to a parent department and yet must belong in effect to another department. Not all teachers would like that type of arrangement and we would not like all teachers to be in the program. Those that feel college prep oriented and find little satisfaction in working with our students would not fit into the plans that we have.

What we are trying to do is to look at tomorrow as best we can and to get students ready for it. We as occupational educators must consider something that Dr. Neil Sullivan, the Commissioner of Education of Massachusetts recently said, "We are in an age where we can no longer bury our mistakes in the labor market." They just won't accept them!

INDUSTRIAL PREPARATORY PROGRAM

SOPHOMORE YEAR

HACKENSACK HIGH SCHOOL
HACKENSACK, NEW JERSEY
07601

HACKENSACK HIGH SCHOOL
HACKENSACK, NEW JERSEY

INDUSTRIAL PREPARATORY PROGRAM

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The Board of Education, the school administration, and the teaching staff are deeply grateful for the valuable assistance provided by the Vocational Division of the New Jersey State Department of Education in the development of this program. A special sense of appreciation is extended to Dr. Morton Margules, Associate State Director of the Vocational Division for the inspiration and support that he contributed to Hackensack High School.

SHOP MATHEMATICS I

TECHNICAL MATHEMATICS I

UNIT I

DUCTWORK

Technical Mathematics I: Introduction

The Technical Mathematics course, as part of the Industrial Prep program, is geared to teach the basic concepts of measurement, their applications, and the related mathematical concepts and skills. Basic topics from arithmetic, algebra, plane and solid geometry, and trigonometry are taught as they are related to measurement. The full year is devoted to measurement as preparation for further study in this three year technical program. A strong understanding of measurement will be necessary for the students when they study the more abstract concepts of units of measurement in the eleventh grade physics course.

During the year the teacher develops the concepts of units of measure for length, angle, area, volume, and weight. Each of these concepts is developed as concretely as possible with great stress placed upon students making their own constructions to illustrate the properties and function of each unit of measure. The students experience the use of an arbitrary unit for each type of measurement. The teacher helps develop appreciation for the controversy caused when two groups of people do not accept the same standard units.

The heart of the course lies in the development of four projects which delve into the various aspects of measurement. The projects are: models of ductwork, geometric solids, densities of metals, and paper box fabrication. Each of these projects has been adopted because it provides a means of teaching the important mathematical concepts which are related to measurement. Each project can be readily adapted to the employment of manipulative skills which have an innate interest to the type of student in the course. The teacher creates a training atmosphere similar to that in a shop-training program in which each apprentice is expected to produce as he learns. In the situation the teacher can adapt to individual differences depending upon the level of competence of each student and, hopefully, help each student develop to his potential. The projects lend themselves to both individual and group contributions in such a way that each student can meet success.

How will it be taught?

Each project will be preceded by several topics which contain background knowledge for the completion of the project. Great stress is placed, throughout the course, upon physical construction of measuring tools and projects by each student. Each construction is used as a basis for further learning. The basic theorems of geometry pertaining to the project are taught using a laboratory approach, in which construction are made by teacher and class together. When developing a geometric construction, the students use their tools of construction in applying their knowledge of measurement. The Geometric theorems related to the construction are learned through the experiences of the students.

The teacher anticipates that the first attempts at construction will be unsatisfactory. He uses the first attempts to help students analyze the basic mathematical relationships which affected the construction. In abstract mathematical properties they had studied to produce successively better manually-constructed projects.

The teacher introduces each topic by using very concrete, explanations. The instructor relates the topic to practical industrial applications, indicating some ramifications of the concepts involved. The introductory terminology is elementary; the teacher avoids the use of definitions until the class has had sufficient experience with the concepts under consideration. Then, with the help of the English teacher of the program, the mathematics teacher delves into the significance of the parts of each definition.

As the topic is developed, the teacher builds the terminology and the sophistication of his approach. At the same time, he helps the students develop their level of understanding and ability to analyze their mistakes. An atmosphere is created in which the students use new vocabulary and concepts in their discussion. They are expected to keep notes in an organized manner and from there on they are expected to apply their knowledge to later projects.

The Technical Mathematics I course is developmental in format and follows a "spiral" presentation of topics and review. Through a laboratory approach, students gain experience in applying a variety of basic mathematical concepts related to the applications of measurement.

Linear Units of Measure

The presentation of each topic dealing with measurement in Technical Mathematics I follows a "spiral" format. In developing the concept of linear measurement, the teacher first develops the process of comparing line segments by superposition and then by copying one distance with a compass and comparing the compass setting with the other line segment. The students gain experience in the techniques of comparing line segments under the guidance of the teacher.

Next, the instructor chooses an arbitrary line segment as a unit of length and uses this as a means of comparison. For example, one line segment is considered longer than another if more units fit end to end in the first line segment than in the second. The students take this unit and construct their own number scales, including fractional parts, and then use the scales to measure line segments.

The instructor then points out that standard units of length have already been accepted by whole nations of people. He "introduces" the units of the English system and pays special attention to the inch and the fractional parts of the inch. He again has students measure with the English units and helps them analyze situations in which the various units would be most conveniently used.

Following this spiral procedure, the teacher introduces the units of the metric system and has his students use these units to measure line segments. Finally, the students measure the same line segments in the English system in order to establish the basic ratios between the units of the two systems.

The basic concepts of linear measurement are applied to the construction of plane geometric figures and to the inductive development of basic geometric theorems. The relationships between various parts of constructed figures are checked by measurement to be certain that the constructions have been done correctly. In this way, the students experience the various functions of measurement as they apply to basic geometric theorems.

UNIT I

Lesson 1.

I. General discussion for need of mathematics.

  A. Mathematics used at home and in play.
    1. By parents.
    2. In summer jobs.
    3. In industry.

  B. Discuss correlation of Industrial Prep program with other departments.
    1. Mathematics, drafting, science, and English.

II. Introduction to projects.

  A. Ducts
    1. Discuss individual contributions.
    2. Discuss cooperative contributions.
      a. Relate to industrial methods, i.e., need for individual and group contributions.

  C. Paper boxes or containers.
    1. Made of box board.
    2. Made of corrugated.
      a. Precision of fabrication.

  D. Weights and volumes.
    1. Display project showing pieces of bar stock which had been cut to desired weight.
    2. Display raw materials used in making project.
      a. Discuss involvement of mathematics in project.

  E. Drawings and layouts from drafting classes to be on display on bulletin board.

Lesson 1 (cont.)

III. Tools required for the year.

   A. Straight edge, pen, pencil, compass, protractor.

   B. Notebook.

Assignment: Procure tools, notebook, and cover textbooks.

Lesson 2.

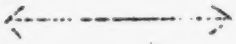
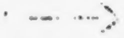
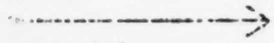
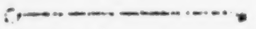
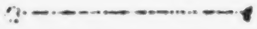
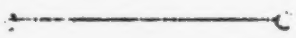
I. Requirements of course.

   A. Completion of assignments.
      1. Course is based on premise that assignments are products of our industry.

II. Notebook requirements.

   A. Most classwork cannot be found in our textbook. Thus it is necessary to keep a complete notebook.
      1. Pages must be numbered and dated.
      2. Homework section separate from note section.
      3. Notebook counted as part of grade.
         a. Notebooks checked periodically, usually during tests.
         b. Vocabulary (special section).

III. Basic geometric symbols and terms.

   A. Understanding basic linear symbols.
      1. Point •
      2. Line 
      3. Half-line 
      4. Ray 
      5. Line segment 
      6. Open line segment 
      7. Half-open line segment 

   B. Vocabulary
      1. Write definitions for above terms using concepts of modern terminology.

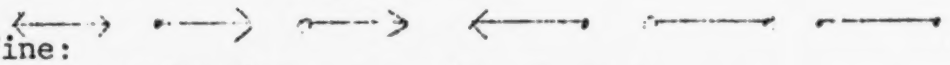
Lesson 2 (cont.)

- a. A point is an exact location in space.
- b. A straight line is the set of points determined as the path of a point which moves in one fixed direction.
- c. Two half-lines are created by removing one point from a line. Each half-line extends indefinitely in one direction only and does not include the point that separates the line into two-half lines.
- d. A geometric plane is a set of points which form a flat surface. A plane has length and width, but no thickness.
- e. A ray is half-line with the point of separation as an endpoint.
- f. A line segment is a definite part of a line, including two endpoints.
- g. An open line segment is the figure formed by removing the endpoints of a line segment.
- h. A half-open line segment is the figure formed by removing only one endpoint of a line-segment.

Assignment: Study and memorize the new terms, symbols, and definitions.

Lesson 3.

I. Basic geometric figures.

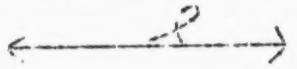
- A. Quiz
  - 1. Identify the following symbols.
    - a. 
  - 2. Define:
    - a. line, half-line, ray, line segment.
- B. Review
  - 1. Geometric terms and symbols.
  - 2. Definitions.
  - 3. Keep notebooks up-to-date.
- C. Linear geometric figures and their properties.
  - 1. Point - has no measure, no thickness.
  - 2. Line - infinitely long, cannot be measured, no thickness.

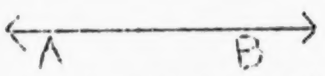
Lesson 3 (cont.)

3. Half-line - relate to line.
4. Ray - relate to line and half-line.
5. Line segment - a definite part of a line, has a given length, has endpoints.

II. Identification of linear geometric figures and use of symbols.

A. Point
   1. Labeled with a capital letter.

B. Line
   1. Labeled with one small letter or two capital letters.
      a. 

read "line L".
      b. 

read "line AB" or "line BA".
         i. Using symbols: $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$
         ii. Note two names for the same line segment:

$$\overleftrightarrow{AB} = \overleftrightarrow{BA}$$

C. Line segment
   1. Label endpoints with capital letters.
      a.

read "line segment AB" or "line segment BA". Using symbols: "$\overline{AB}$: or "$\overline{BA}$"
      b. A line segment can be named in two ways:

$$\overline{AB} = \overline{BA}$$

III. Relationship between two linear geometric figures.

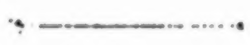
A. Points
   1. No dimensions.
   2. Used only to indicate location.

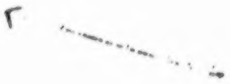
B. Lines
   1. Infinitely long.
   2. Infinite set of points as the path of one point which goes in specific direction.

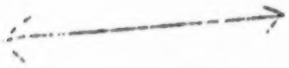
Name ____________________ Date __________

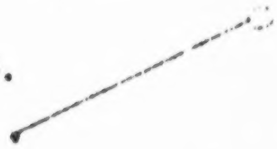
Quiz (Lesson 3)

For each problem, identify the symbol given.

1. 

2. 

3. 

4. 

For each problem explain the meaning of the terms.

5. Ray

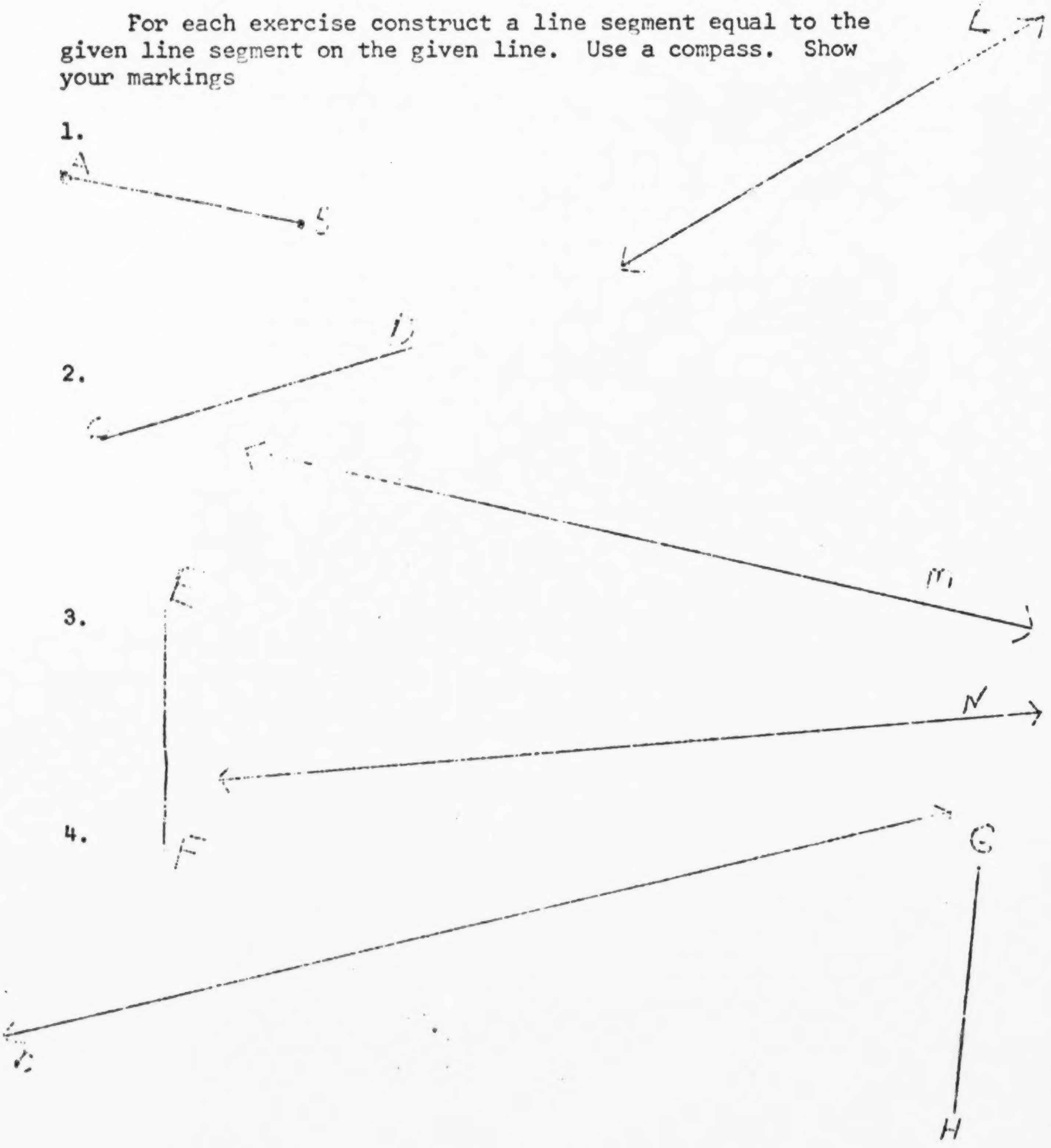
6. Point

7. Half-open line segment.

Name____________________ Date___________

Assignment (Lesson 3)

For each exercise construct a line segment equal to the given line segment on the given line. Use a compass. Show your markings



C. Line segments
   1. Each is part of a line, but has endpoints.

IV. In what ways can two line segments be compared?

A. Line segment is part of a line, but has endpoints.

B. $\overline{AB} = \overline{CD}$ if, without stretching $\overline{AB}$, it is possible to make point A coincide with point C, points of $\overline{AB}$ can be made to coincide with $\overline{CD}$ and, as a result, point B coincides with point D.

   1. Demonstrate using a tracing of one line segment.

C. To compare in an easier manner, set the points of a compass on the endpoints of AB: then, with the same setting, place one point of the compass on C and observe whether the other compass point will coincide with point D.

   1. To compare two line segments to tell if they match, use compass as follows:
      a. On blackboard, construct three or more line segments, and label them. Then construct equal number of lines, label with small letters. Have students come to the board and reproduce given line segments on each line.
      b. Speak of a 1-1 correspondence between the line segment and the line segment constructed, and the line segment determined by the points of the compass.

D. Discussion: In how many ways can two line segments be compared for matching?
   1. By the method of matching up endpoints and comparing the location of the other endpoints.
   2. By using a compass setting or third line segment.
   3. By comparing each line segment with a standard scale, or markings on a ruler.

E. What do we mean when we say "measure the line segment AB"?
   1. We compare the line segment AB with a standard number scale.
      a. We shall discuss this later.

Lesson 3 (cont.)

Assignment: Study new concepts and terminology in notes. On hectograph paper construct three line segments which are equal to each of the three given line segments on the hectograph paper.

Lesson 4

I. The number line

   A. Quiz
      1. Identify the following geometric terms with their appropriate symbols.
      2. Reproduce line segment AB on line m .

   B. Review to date.
      1. Geometric terms and symbols, definitions.
      2. Reproducing line segment.
      3. Equality of line segments.

   C. Introduction of a unit of measure and the number line.
      1. Given line segment AB, and line m .
         a. Choose an appropriate point on the line m and call it "Zero", 0.
         b. Mark off to the right of 0 a line segment equal in length equal to $\overline{AB}$. Call the other endpoint "1".
         c. Continue placing the unit end-to-end and locate new endpoints, calling them 2, 3, 4, . . .
         d. If a student has not submitted the remark that "could we make the number line to the left of zero , encourage them to observe same.

            i. Construct number line to the left of zero and decide on negative symbol for numerals.

      2. Class discussion on the distance between any two consecutive points on the constructed number line.
         a. This distance is called a unit distance.

   D. Pass out assignment sheet containing unit distance $\overline{AB}$ and line L with point 0 marked on line L.

Name________________________ Date________

Quiz (Lesson 4)

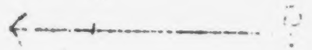
For each problem write the symbol which represents the given geometric term.

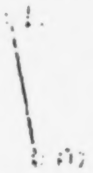
1. "Line segment KL".

2. "Half-open line segment NM".

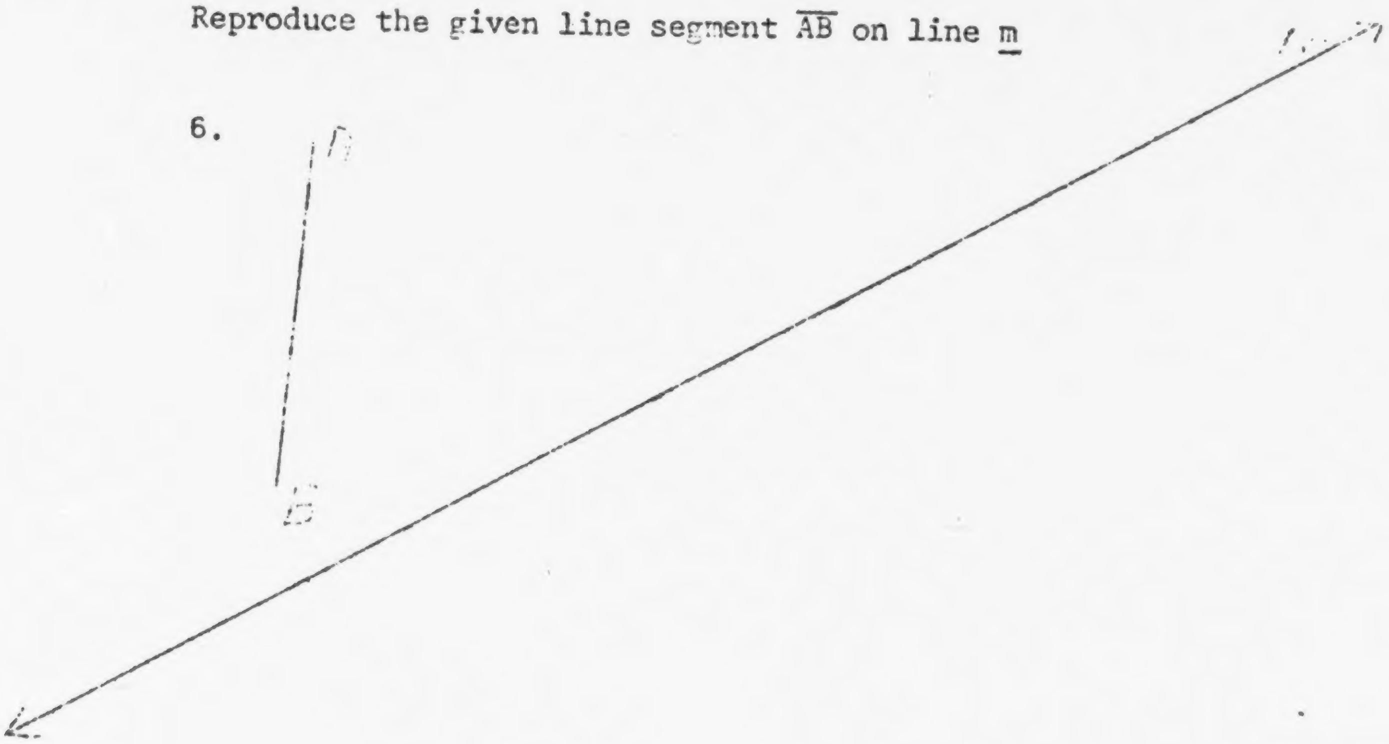
3. "Ray TU".

For each problem name the geometric term represented by the given symbol

4. 

5. 

Reproduce the given line segment $\overline{AB}$ on line m

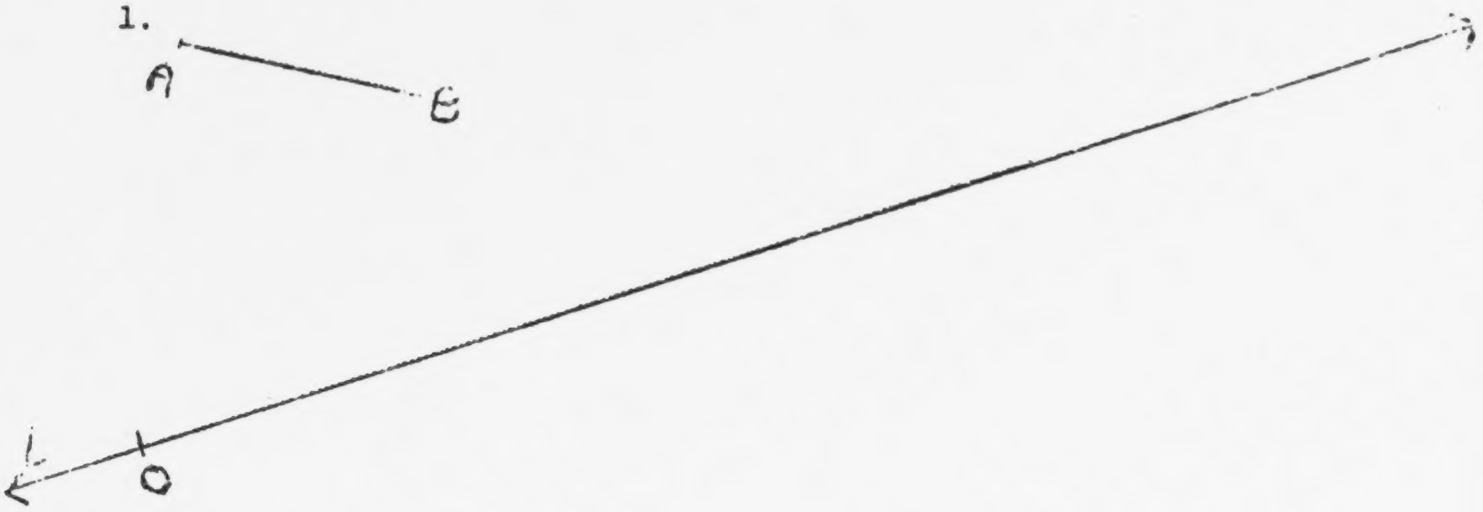
6. 

Name____________________ Date __________

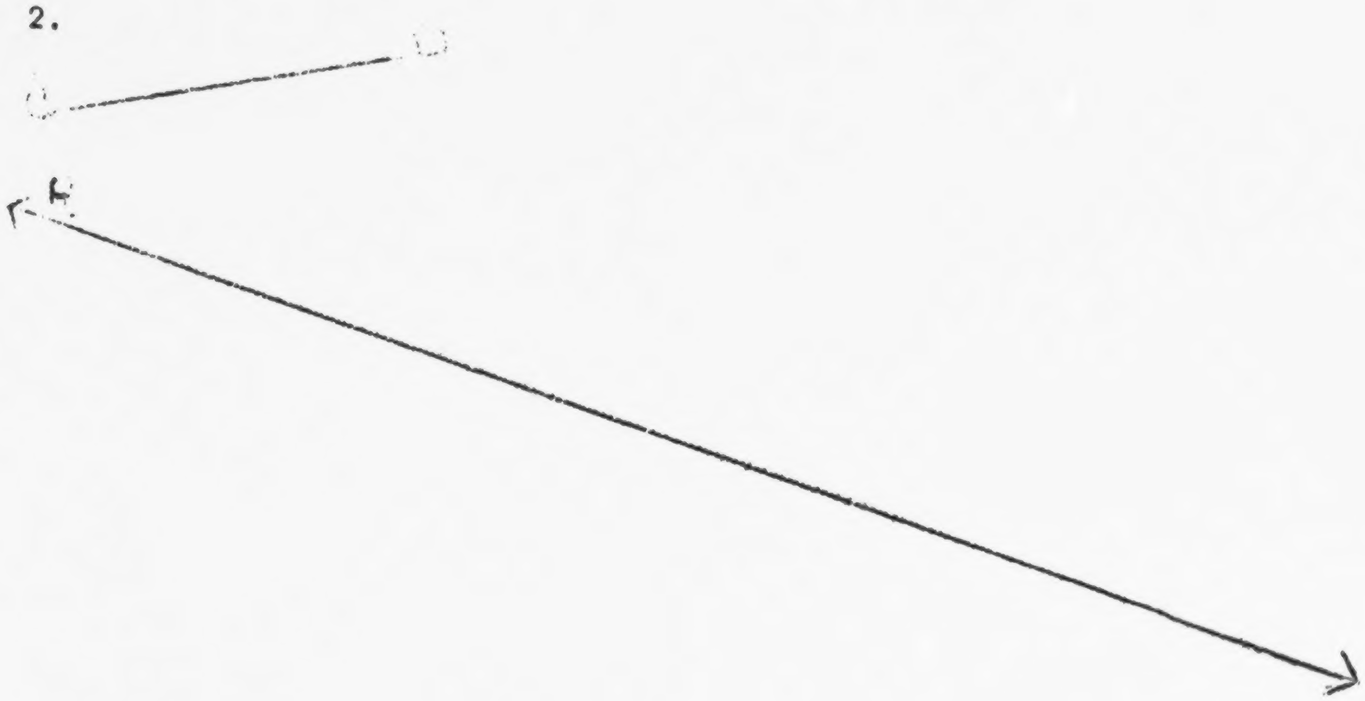
Classwork Assignment (Lesson 4)

For each exercise construct a number line on the given line using the given line segment as a unit distance. Begin at point 0" as the zero point.

1.



2.



Name ____________________ Date ____________

Assignment (Lesson 4)

For each exercise construct a number line on the given line using $\overline{CD}$ as a unit distance. Begin at point "0" as the zero point.

1. Develop a number line in unit lengths.

2. Develop a number line in half-unit lengths.

3. Develop a number line in quarter-unit lengths.

4. Develop a number line in eighth-unit lengths.

Lesson 4 (cont.)

  1. Students construct a number line.

II. Bisect a line segment.

  A. Compass and straight edge.
     1. Classwork and demonstration.

  B. Application to number line.
     1. Construct number line using given unit, then bisect each unit length.
     2. Repeat to form quarter units.
     3. Repeat to form eighth units.
     4. Discuss need for accuracy and neatness to obtain good result.

Assignment: On new assignment sheet, follow steps of part II above.

Lesson 5

I. Using a number line.
  A. Review of bisection of line segments.

  B. Construction of number line, given arbitrary unit of length.
     1. Construct scale using units end-to-end, then bisect each segment.
     2. Bisect the given unit first, then mark off each half-unit on the given line.

  C. Ordering of numbers on a number line; class work.
     1. Approximate the position of points on a given number line which correspond to each of the following numbers.

        a. 5 , 1/2 , 0 , 2, 13/4 31/4, 32/8

        b. 3/8 ,2 3/4 ,2 1/4,2 2/8 ,3 3/8 ,1 6/8

        i. Note that 2 1/4 and 2 2/8 are names for the same point on the number line.

  D. Introduce the unit of measure "1 inch".

Lesson 5 (cont.)

1. Classwork on assignment sheet.

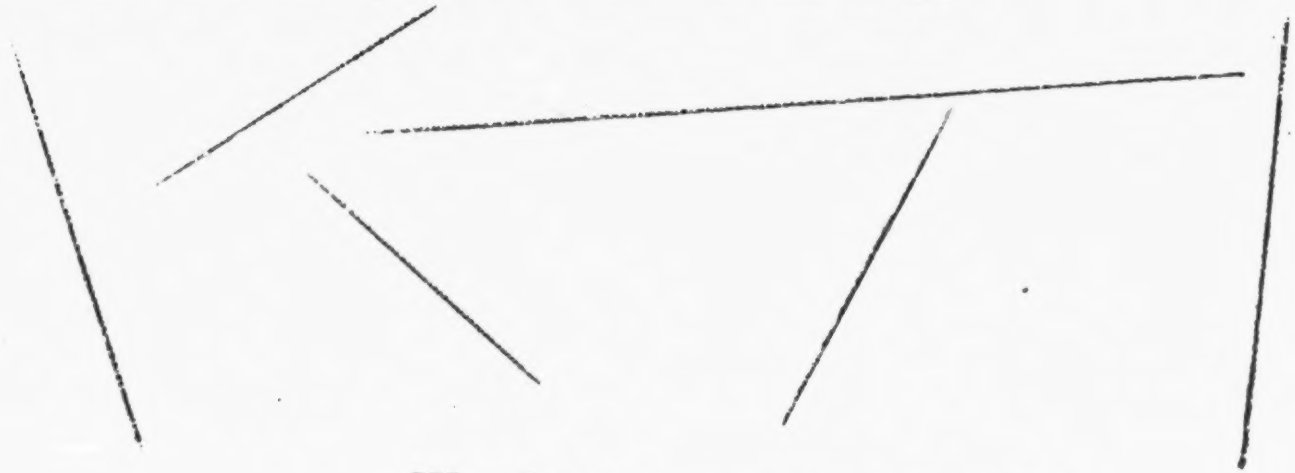
Name ____________________ Date __________

Assignment (Lesson 5)

For each exercise, fill in the table at the bottom of this page.

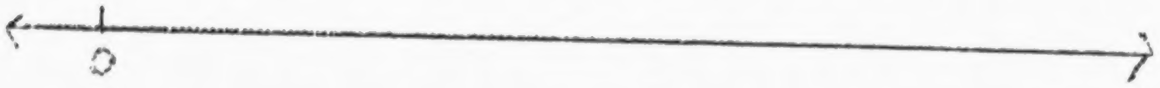
1. Construct a number line on line L using one inch as the unit. Use point "0" as the zero point. Refer to the line segments through F . Estimate the length of each line segment to the nearest inch. Record your estimates in the first row of the table.

II. Measure each of these line segments to the nearest inch. Record the results in the second row of the table.



III. Construct a number line in half-unit parts on line M using one inch as the unit. Use point "0" as the zero point.

Now estimate the length of each line segment above to the nearest half-inch. Record your estimates in the third row of the table.



IV. Measure each of the line segments to the nearest half-inch. Record the results in the fourth row of the table.

Lesson 5 continued

Technique of measurement	A	B	C	D	E	F
Estimate: nearest inch						
Measure: nearest inch						
Estimate: nearest half inch						

Lesson 5 (cont.)

a. Unit of measure given, line L; construct number line.
b. Measure line segments on assignment sheet to nearest inch.
c. Measure same line segments to nearest half inch.

Assignment: Repeat steps in part D on new assignment sheet.

Lesson 6

I. Introduction to the English system.

A. Quiz
  1. Order the following numbers on a sketch of a number line.
  2. Given $\overline{AB}$ equal to one inch. Using compass, determine the length of the following line segments to the nearest 1/4 inch.

B. Review homework

C. Introduce English system of measurement.
  1. Identify the different units of measure
    a. Inch (standard unit of measure)
    b. Foot = 12 inches
    c. Yard = 3 feet = 36 inches
    d. Mile = 5,280 feet
  2. Each of the above may be considered as a unit of measure.
  3. Discuss situations in which these units are used.
    a. Which unit should be used to measure:

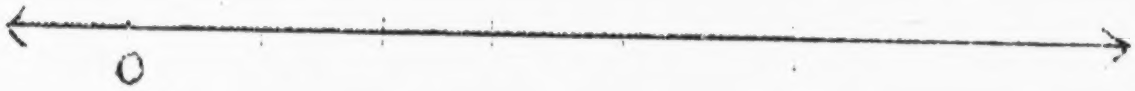
      i. the length of my shoe.
      ii. the height of the room.
      iii. the distance from here to New York.
      iv. the length of a street.
      v. the length of a pencil.
      vi. the width of a pencil.
      vii. the distance to the moon.
      viii. the thickness of a fingernail.

Name ____________________ Date ________________

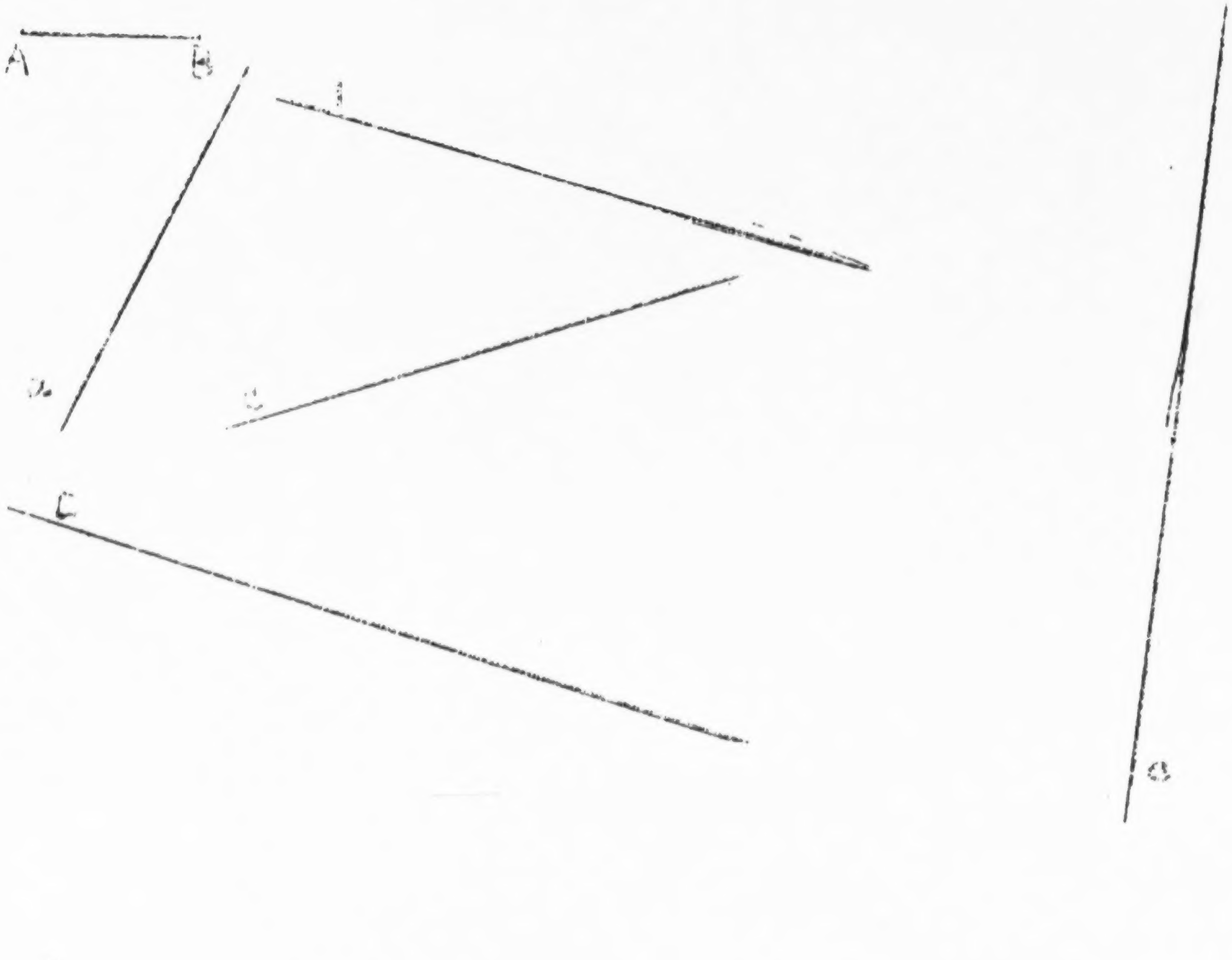
Quiz: Lesson 6

1. Locate the following numbers on the given sketch of a number line.

2 1/4, 0, 1/8, 1 3/8, 3 7/8, 1 5/8, 4 1/4

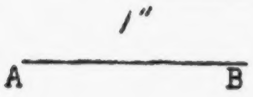


2. Given $\overline{AB}$ equal to one inch. Using compass, determine the length of the following line segments to the nearest 1/4 inch.



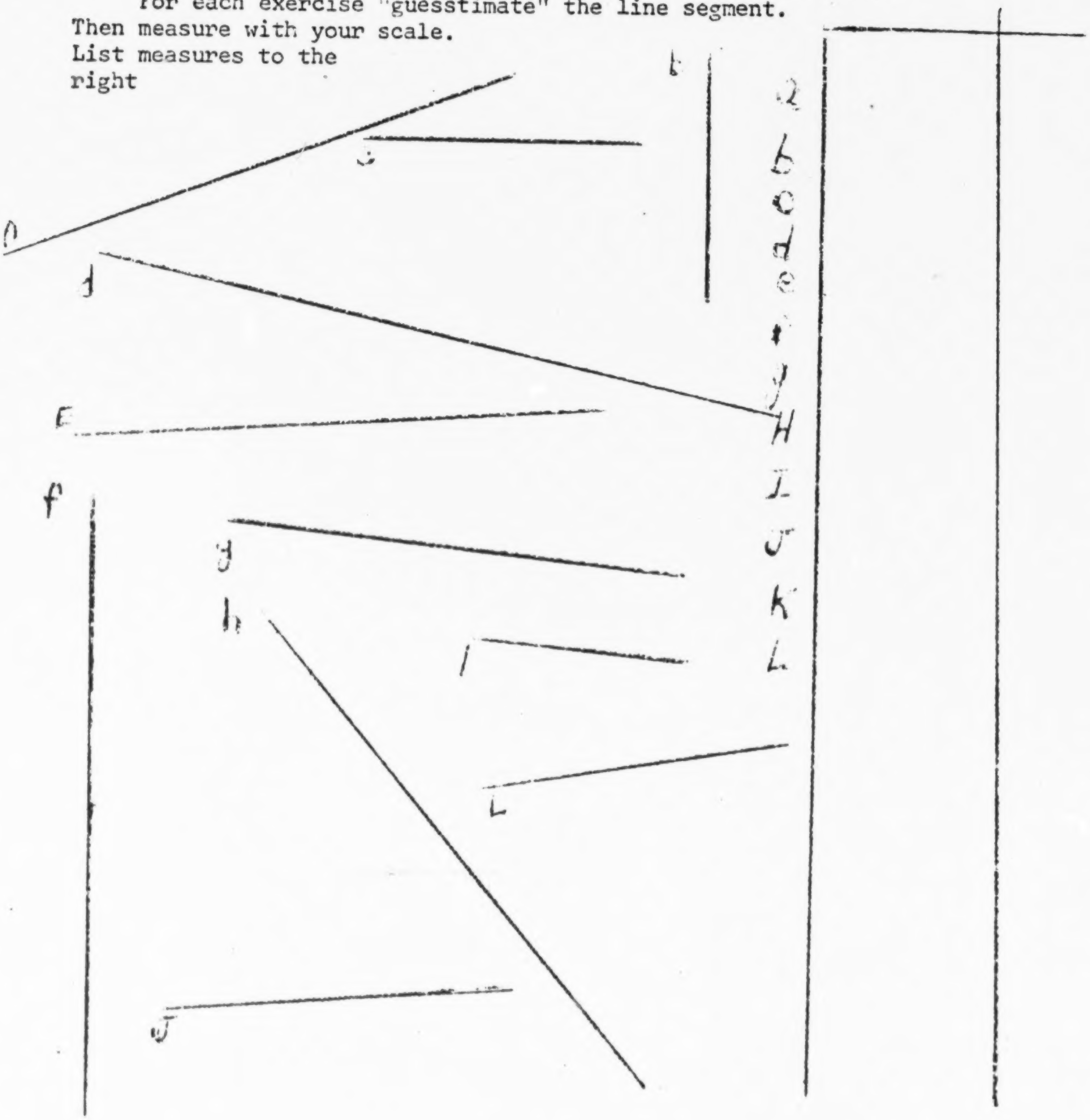
Name ____________________ Date ____________________

Classwork assignment: Lesson 6



Construct a 12" ruler on cardboard. Make the scale to 1/4".

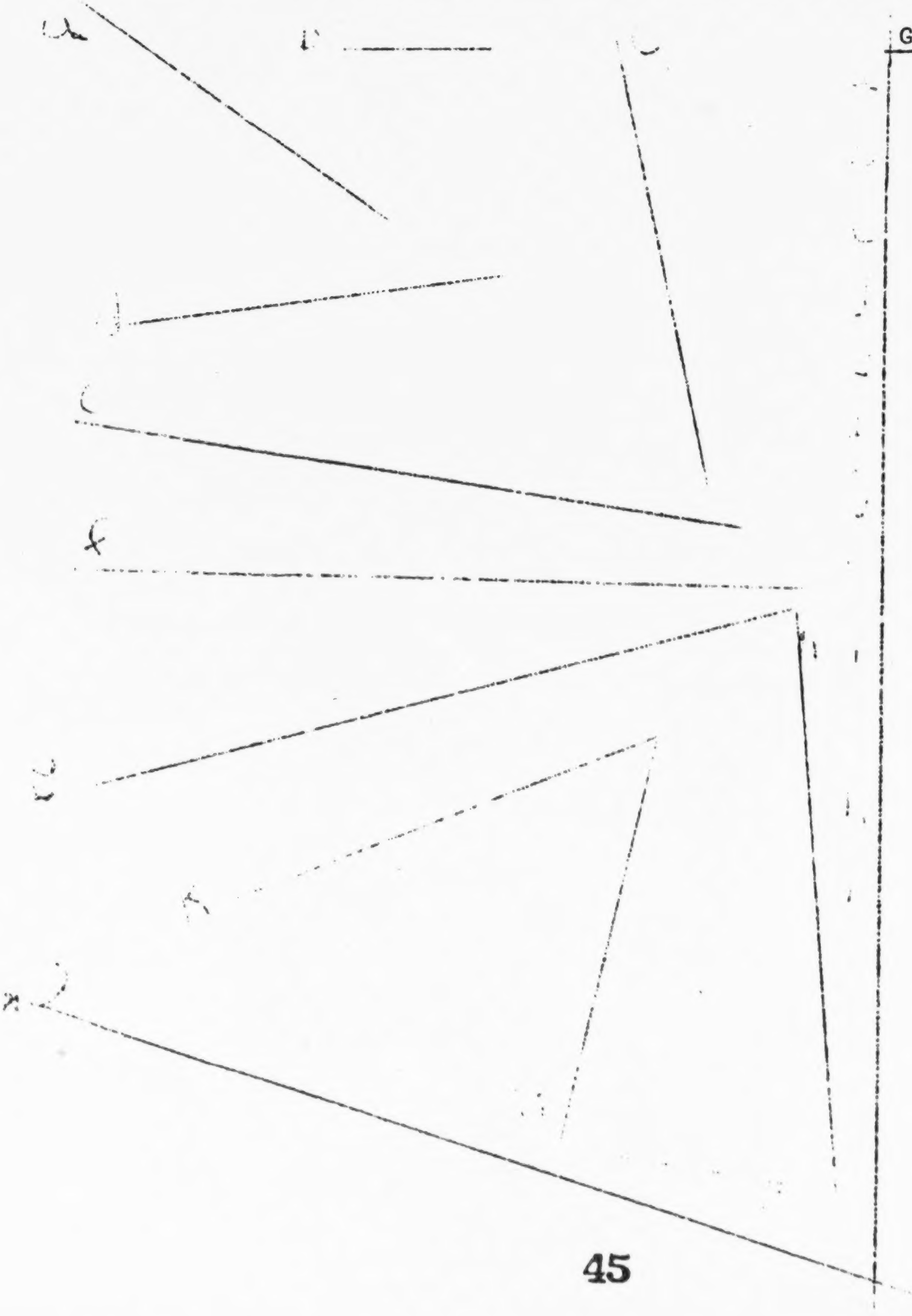
For each exercise "guesstimate" the line segment. Then measure with your scale. List measures to the right



Name ___________ Date _________

Assignment: Lesson 6 $\underset{A\qquad\qquad B}{\overline{\quad 1'' \quad}}$

For each exercise "guesstimate" the line segment to nearest 1/8". Then measure with your scale. List measures to the right.



Guess	Measure

Lesson 6 (cont.)

  b. Discuss advantages of larger units of measure and smaller units of measure.

    i. Size of numeral used, convenience of measurement, convenience in calculating.
    ii. Convenience in using integral measures.

D. Classwork on estimating lengths of line segments using inch as the unit.

  a. Meaning and purpose of estimating.

    i. To approximate the length of the object.
    ii. To check the result of an actual measurement.
    iii. To develop a better understanding of measurement.

Assignment: Hectographed page, estimating and measuring line segments using a 12" ruler containing 1/4" scale on cardboard.

Lesson 7

I. Introduction to the metric system.

  A. Review different units of measure in English system.
    1. Uses of each unit in two or three examples.
      a. Discuss measurement of desk top in three different units of measure: inch, foot, yard.
      b. Measure length of desk top in miles.

  B. Introduce centimeter as unit of measure.
    1. Construct ruler (12 inches long) using cm. as unit.
    2. Compare number of cm. in ruler to 12 inches and number of centimeters.

  C. Develop concept of cm. as compared to inch.
    1. Assignment sheet containing various line segments. (in integral multiples of an inch).

Lesson 7 (cont.)

a. Measure each line segment to nearest inch.
b. Measure each line segment to nearest half-cm.
c. Compare corresponding measurements.

Assignment: On hectographed page, repeat part C above.

Lesson 8

I. The metric system

A. Quiz
   1. Measure given line segments in cm., using your own cm. scale.
   2. Approximate each line segment for length in inches using results in step 1 as an aid.

B. Review homework.
   1. Stres comparison of line segments in inches and cm.
   2. Number of cm. in one foot (approximate).
   3. Number of cm. in one inch (approximate).

C. Conversion: inches to cm., cm to inches (use 1 inch = 2.5 cm.).

   1. Using scale on overhead projector:
      A. Convert 3 inches to cm.
      b. Have class convert following to cm. :

         i. 8", 25", 17', 6 1/2", 140 inches.
         ii. Note that number of cm. exceeds number of inches in each measurement.

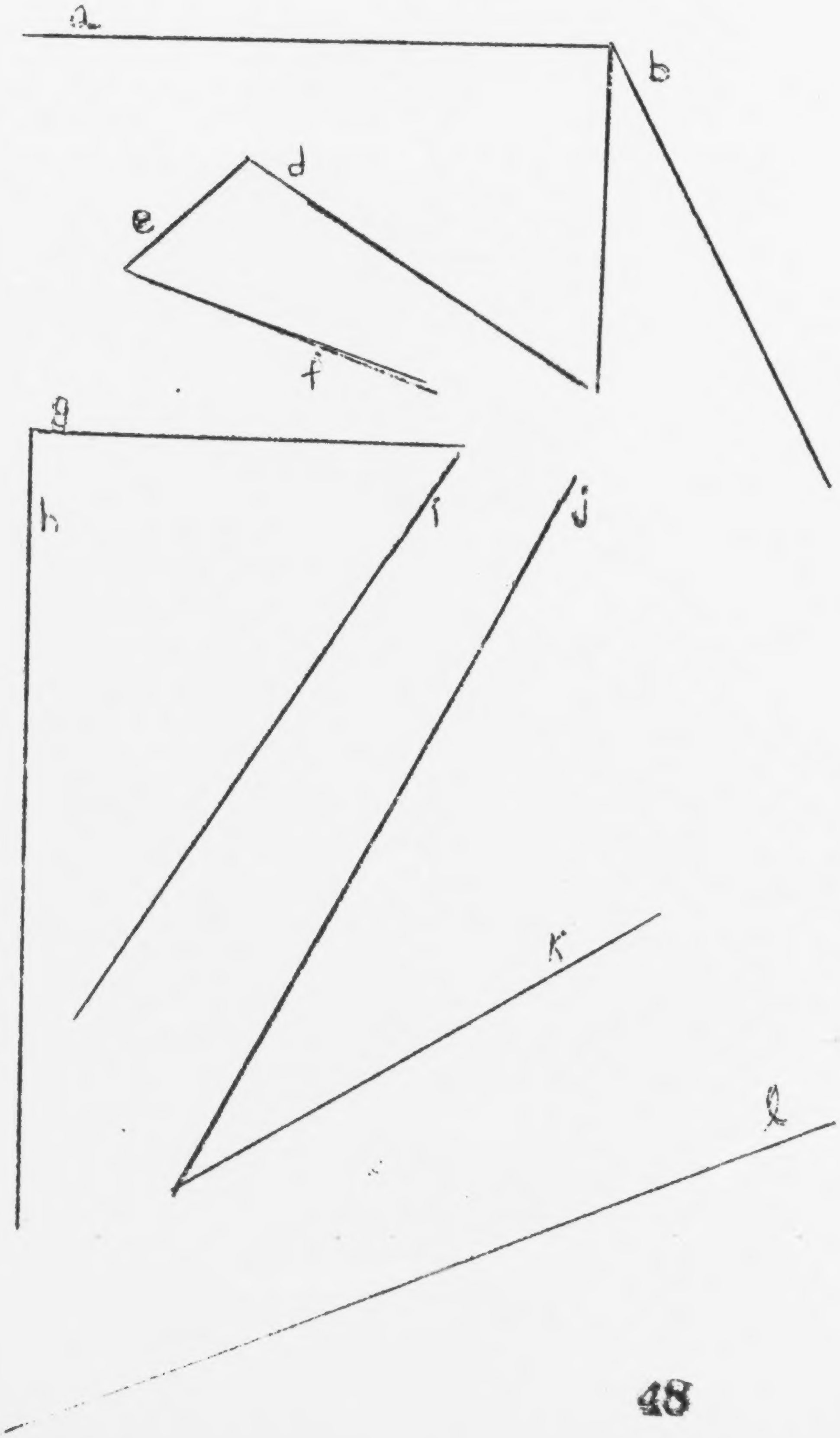
   2. Using scale on overhead projector:
      a. Convert cm. to inches (nearest inch).

         i. 25 cm., 60 cm., 900 cm., 35 cm., 165 cm.
         ii. Note that number of inches is less than number of cm.

Name ____________________ Date _____________

Classwork Assignment: Lesson 7

a) Measure each line segment to the nearest inch.
b) Measure each line segment to the nearest 1/2 cm.
c) What generalization can you make showing the relation between the number measure in inches and in centimaters?

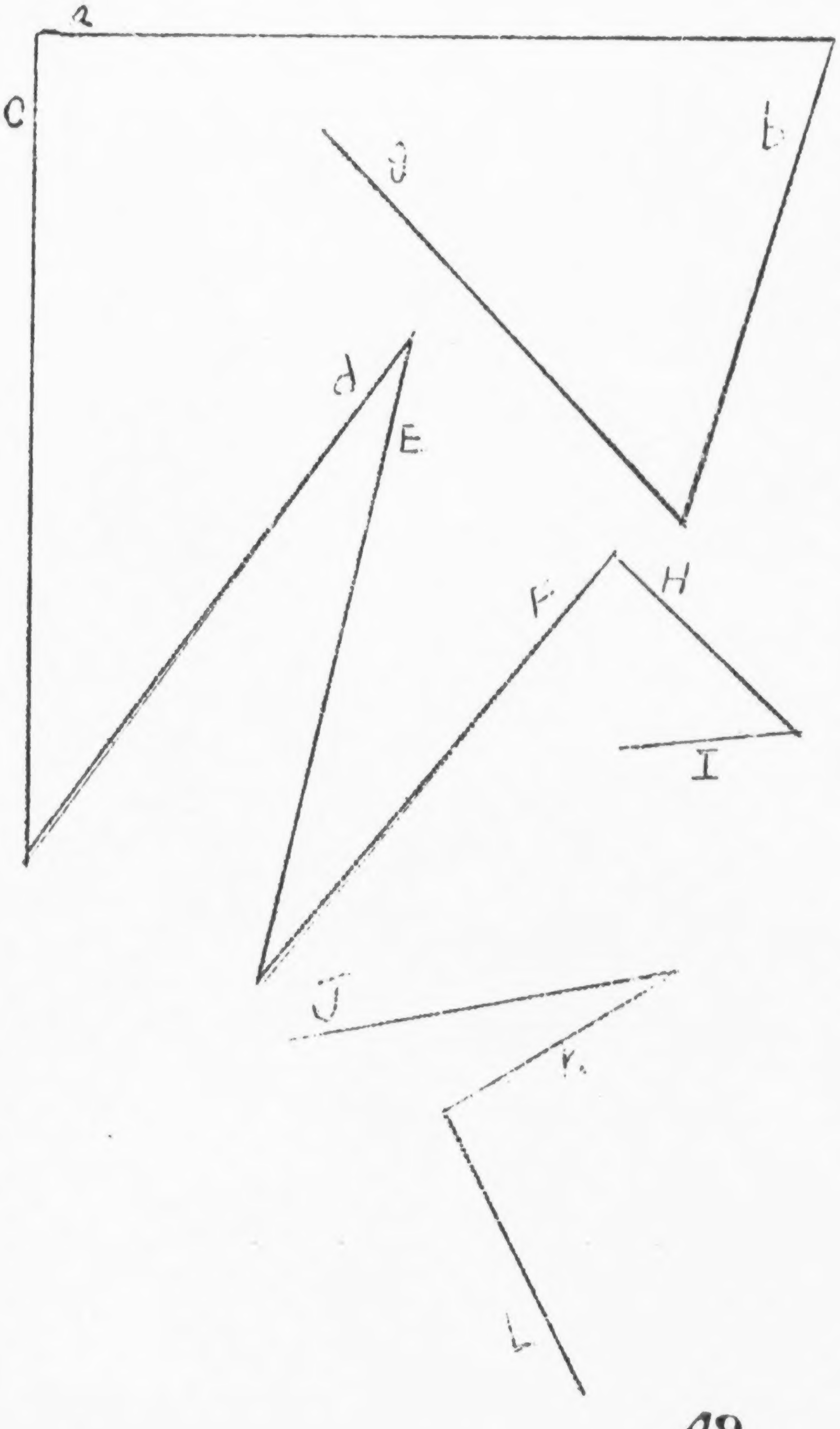


	Measure	
	Inches	Centimaters
a		
b		
c		
d		
e		
f		
g		
h		
i		
j		
k		
l		

Name ____________________ Date _______________

Classwork Assignment· Lesson 7

a) Measure each line segment to the nearest inch.
b) Measure each line segment to the nearest 1/2 cm.
c) What generalization can you make showing the relation between the number measure in inches and in centimeters?

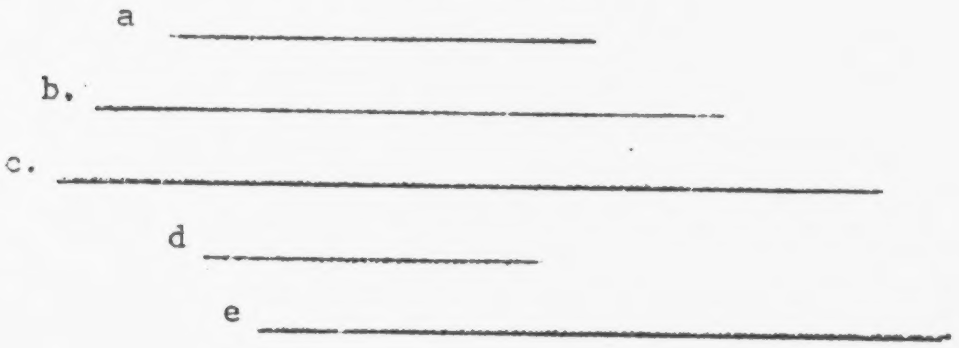


	Measure	
	Inches	Centimeters
A		
b		
c		
D		
E		
F		
g		
H		
I		
J		
k		
L		

Name ____________ Date ____________

Quiz: Lesson 8

Using your own scale, measure the given line segments in centimeters. Then approximate each line segment for length in inches using the results of your measure in centimeters.



	Measure Centimeters	Inches
a		
b.		
c.		
d		
e		

Name ____________________ Date ______________

Assignment: Lesson 8

For each exercise find the reqired conversion.

1. Change 25 Cm. to the nearest inch.
2. Change 2 inches to the nearest cm.
3. Change 35 cm. to the nearest 1/2 inch.
4. Change 5 inches to the nearest 1/2 cm.
5. Change 3 1/2 inches to the nearest 1/2 cm.
6. Change 2 3/4 inches to the nearest 1/2 cm.
7. Change 60 mm. to cm.
8. Change 24 cm. to mm.
9. Change 95 mm. to cm.
10. Change 85 cm. to mm.

Lesson 8 (cont.)

3. Introduce mm. as a unit.
   a. Use 1-foot ruler with metric scale as model.
   b. mm. obtained by dividing 1 cm. into 10 equal parts.
   c. If 1 cm. is divided in half, each half is ________ mm.?
   d. How many mm. are contained in one mm.?
   e. How many cm. are contained in one mm.?
   f. Of the following units of measure, which is the largest in length? cm., mm., inch.
   g. Of the following units of measure, which is the smallest in length? cm., foot, mm., inch.
   h. Spell words represented by following symbols: cm., mm.
   i. Where is it practical to use mm. as a unit; where is it not practical?

Assignment: Convert measures:
Inches to cm., cm. to mm., mm. to cm., cm. to inches.

Lesson 9

I. Comparison of metric units.

   A. Quiz
      1. Change measures in metric to English system.
      2. Change measures in English to metric system.

   B. Review of relationship of mm. to cm.
      1. cm. is 10 times larger than 1 mm.
      2. mm. is 1/10 as large as one cm.

   C. Introduce meter, and relation to other units.
      1. 100 cm. = 1 meter.
      2. ___?___ mm. = 1 meter.
      3. Use meter stick as model: _____inches = 1 meter.

Name ________________ Date ____________

Assignment: Lesson 9

For each exercise find the required conversion.

1. Change 38 mm. to the nearest cm.

2. Change 2 1/2 meters to cm.

3. Change 2 1/2 meters to mm.

4. How many meters are there in 68 cm.?

5. How many meters are there in 68 cm.?

6. A meter is ________ times as large as one cm.

7. A meter is _________times as large as one mm.

8. A kilometer is ______ times as large as one meter.

9. Tne kilometer is equal to _______ meters.

10. Ten kilometers is equal to __________mm.

Lesson 9 (cont.)

D. Kilometer, and relation to other units.
   1. km. = 1,000 m.
   2. km. = _____ cm.
   3. km. = _____ mm.
   4. note: relation in multiples of 10.

E. Conversions (classwork): change:
   1. one km. to meters.
   2. 10 km. to meters.
   3. 35 km. to meters.
   4. 35 km. to mm.
   5. 350 cm. to meters (nearest half-meter).
   6. 2,500 cm. to nearest half-meter.
   7. 2,500 cm. to mm.
   8. 2,500 meters to mm.
   9. 2,500 meters to cm.

F. Reorder from largest unit to smallest.
   1. meter, kilometer, centimeter, millimeter, inch.

Assignment: Study today's notes. Review for test tomorrow.
Do set of conversion problems.

Lesson 10

I. Comparison of English and metric units.

   A. Quiz
      1. Conversion of measures in metric system.

   B. Review km., m., cm., and mm.
      1. Largest to smallest in unit length.
      2. Multiples of 10.
      3. Use meter stick for comparisons.

   C. Comparison of English and metric system.
      1. 1 inch is approximately 2.54 cm.
      2. 1 inch is approximately 25.4 mm.
      3. One meter is approximately 39.4 inches.

   D. Class problems:
      1. How many feet in one mile?

Lesson 10 (cont.)

2. How many meters in one mile?
3. Which is larger, a yard or a meter?
4. How many yards in one mile?
5. How many meters in one kilometer?
6. Which is a larger unit of measure, 1 mile or 1 kilometer?
7. A kilometer is approximately what part of a mile?

Assignment:

1. A line segment is 15 cm. long. How long is it to the nearest 1/8 inch?
2. A line segment is 10 1/4 inches long.
   How long is it to the nearest cm.?
   To the nearest mm.?
3. Add 15 mm. to 65mm. How many cm. is their sum?
4. Approximately how many feet are there in one kilometer?
5. John walked 350 meters. How many yards did he walk?
6. How many feet are equivalent to 100 meters?

Lesson 11

I. Half-period test.

II. Review test and homework.

Lesson 12.

I. Review test

II. Angles, identification of parts.

A. Definition: an angle is a geometric figure formed by two distinct rays having a common endpoint.

1. Review definition of ray and identification of a ray.

Lesson 12 (cont.)

B. Demonstrate how an angle is drawn and identified.
   1. Draw one ray, called the initial ray.
   2. Draw second ray having same endpoint, call it terminal ray.
   3. Show direction from initial ray to terminal ray by

C. Class constructs angles, following above instructions.

D. Symbol for angle is ∢

E. Classwork: Construct an angle.
   1. Draw $\overline{AB}$, call it initial ray.
   2. Draw $\overline{AC}$, call it terminal ray.

F. Question: do the lengths of the rays affect the angle?
   1. No, since rays are infinitely long, any representation of a ray may be extended indefinitely.

G. Identification of parts of an angle.
   1. Interior, angle (rays), exterior, vertex--(common endpoint).

H. Reproduce a given angle.
   1. Straight edge and compass.
   2. Have class draw any angle.
   3. Repeat construction of a given angle, teacher demonstrating on board.
   4. Have students repeat on own, teacher circulate around class, helping those having difficulties.

Assignment: Reproduce angles given on hectograph paper.

Lesson 13.

I. Measurement of angles.

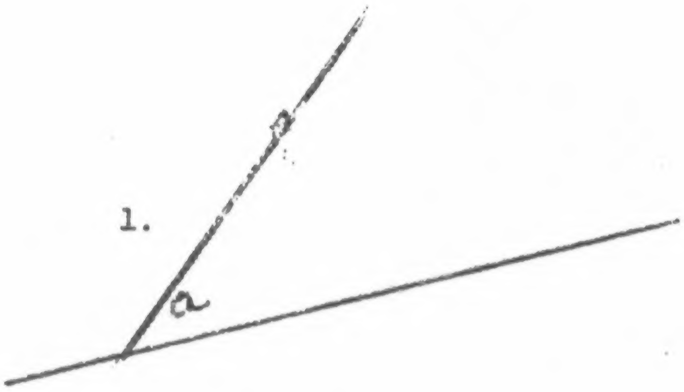
   A. Review of parts of an angle.
      1. Vertex. rays, interior, exterior.

Name ____________________ Date __________

Quiz: Lesson 13

For each problem, reproduce the given angle.

1.



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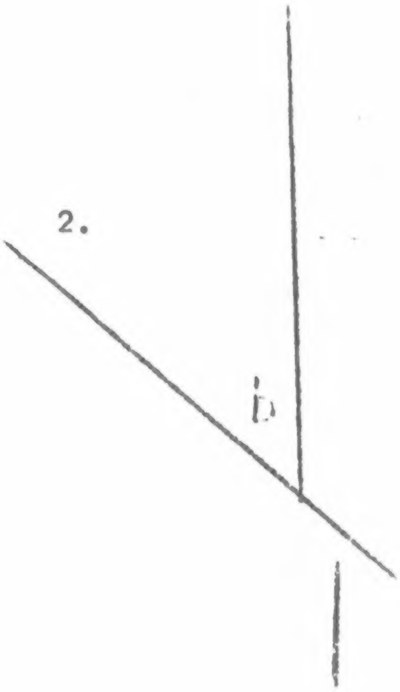
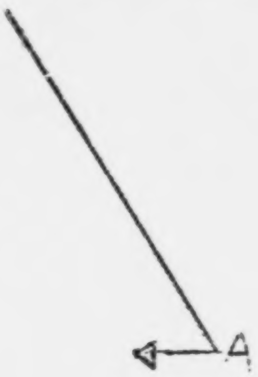
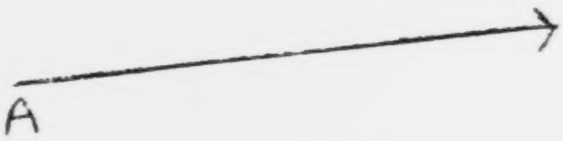


Name ____________________ Date ____________

Assignment: Lesson 13

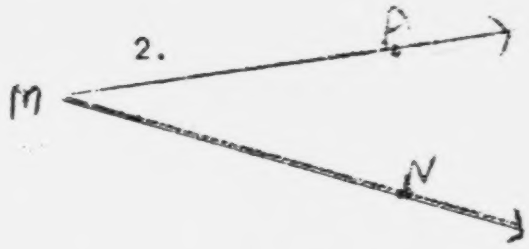
1. Reproduce this angle on the ray to the right.





Name each angle.

2.



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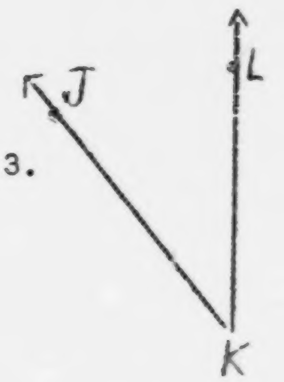

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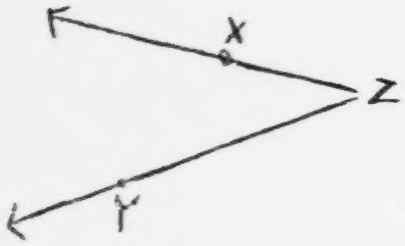
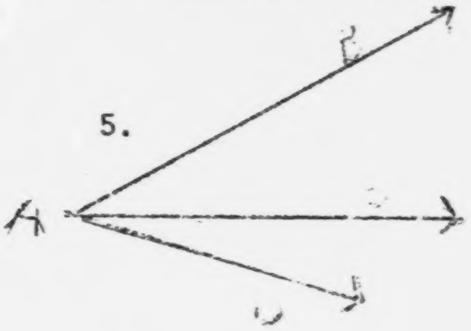


For each exercise, name the pairs of adjacent angles.

5.



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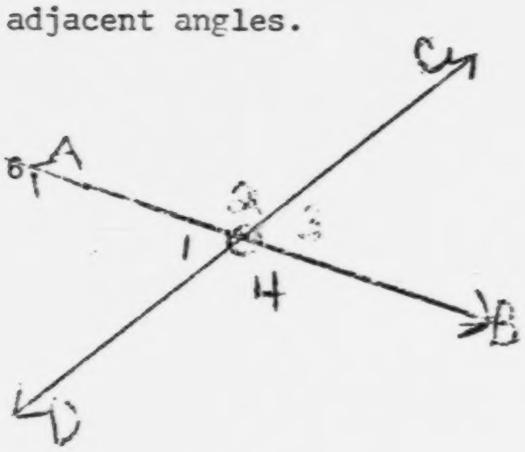

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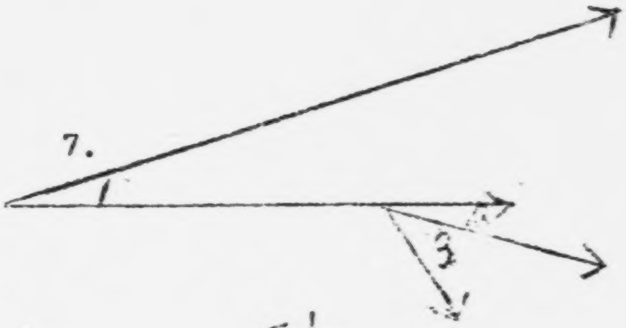

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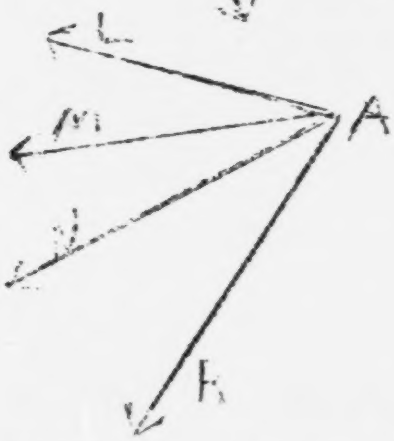
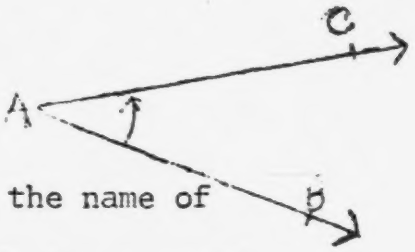


B. Review method of reproducing an angle with straightedge and compass.
   1. Choose one ray as the initial ray.
   2. Then second ray is the terminal ray.
   3. Draw a curved (circular) arrow showing the interior of the angle from the initial ray to the terminal ray.
   4. Have class reproduce two angles on hectograph paper.

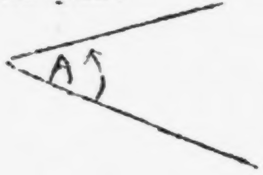
C. Review method of naming angles and parts of angles.
   1. In naming angles the name of the vertex must be written between the names of the other two points.

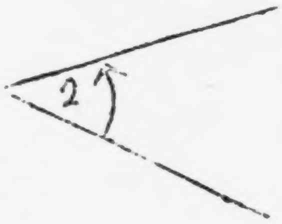
      a. Example: angle BAC: ∡BAC
         or angle CAB: ∡ CAB
         or angle A : ∡ A



   2. Thus, an angle may be named by the name of its vertex.
   3. An angle can also be named by placing a small letter or a small numeral in its interior.

      a. Example:





D. Class participation on board.
   1. Draw an angle, name all its parts.
   2. How many ways can the angle be named?
   3. List the names (use the symbol ∡ for angle).

E. Adjacent angles.
   1. Illustrate on board.
   2. How many angles are in the figure?
   3. Name the angles. (note that "A" is always written in the center).
   4. Can we name any of these angles ∡ A?

      a. Discuss. Note ambiguity.

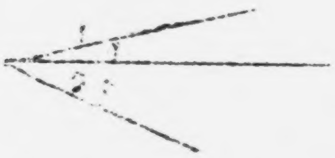
Lesson 13 (cont.)

5. ∢BAC and ∢CAD are called adjacent angles.
6. ∢BAD and ∢BAC are not adjacent angles.

F. Definition of adjacent angles.
   1. Two angles with a common vertex and a common side between them.

G. Examples:

1.



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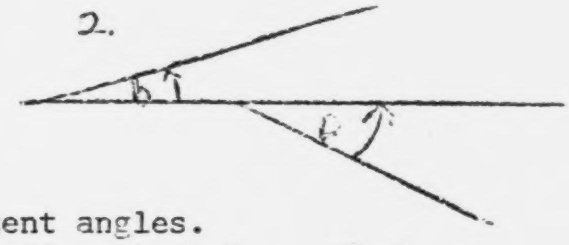
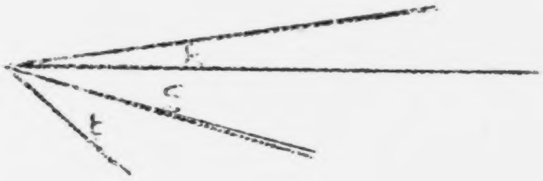


1. ∢ 1 and ∢ 2 are adjacent angles.
2. ∢ a and ∢ b are not adjacent angles. Why?



3. Name the adjacent angles in the above figure.
   a. How many angles are in the figure?
      i. At this time, we are only considering acute angles.

Assignment:

1. Reproduce the given angle.
2. Given sets of angles, name each angle, name the pairs of adjacent angles.

Lesson 14

I. Introduction to angle measurement.

   A. Quiz
      1. Reproduce the given angles.
      2. Name the angle in the figure in two different ways.
      3. Define adjacent angles.
      4. Name two pairs of adjacent angles in the given figure.

   B. Review of adjacent angles.

   C. Bisecting an angle.
      1. By straightedge and compass.
         a. Relate to reproducing an angle.

            i. For bisecting
            ii. Check for equal angles.

   D. Repeat part C as classwork, using hectographed page.
      1. Follow directions of teacher.

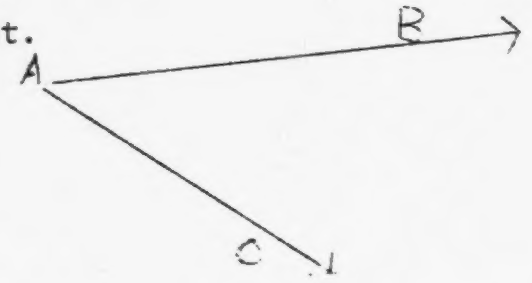
   E. Angle measurement
      1. Meaning of a unit angle to measure other angles.
         a. Relation to unit of length, unit of time.
      2. Use an arbitrary unit of angle measure.
         a. Use hectograph page containing 10 or more angles and an arbitrary unit of measure for angles.
         b. Cut out unit angle from hectographed page and measure angles to the nearest unit.
         c. Discuss this unit angle as an arbitrary unit; similar to the inch and cm.
         d. Indicate that there is an accepted unit of measure for determining the size of angles.

   F. Note: the length of the rays do not affect the size of the angle.
      1. On hectographed page, have each student extend rays of an angle to the edge of paper and then measure the angle a second time.

Name ______________________ Date ____________

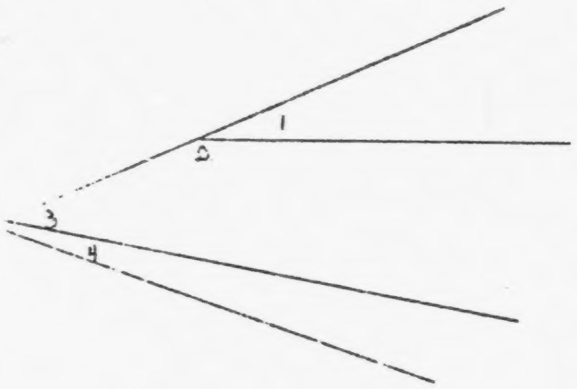
Quiz: Lesson 14

1. Reproduce the angle at the right.



2. Name the angle in problem 1 in two different ways.

3. Define adjacent angles.



4. Name two pairs of adjacent angles in the following figure.

Lesson 14 (cont.)

  a. How does the second measure compare to the first measure?
  b. Conclusion: the length of the ray does not affect the size of the angle.

G. Introduce the basic properties of a circle. Demonstrate on board.
  1. Definition: the set of points which are equidistant from a given point.
    a. A circle is a closed curved line.
    b. The given point is called the center of the circle.

  2. A circle cuts a plane into three regions.
    a. Interior of a circle (contains the center) called a disk.
    b. The circle (a closed curved line).
    c. The exterior of the circle.
  3. Special linear measures of the circle.
    a. Radius: the distance from any point to the center of the circle. (define distance)

      i. Students draw a straight line segment from a point, K, on the circle to the center, P.
      ii. Shortest distance between these two points is the straight line segment PK.

    b. The line segment drawn in (i) is also called a radius.

      i. An infinite number of such line segments can be drawn in a circle.
      ii. Conclusion: Every circle has infinitely many radii.

    c. Diameter: the distance from any point on the circle along a straight line through the center of the circle to the point where the line meets the circle.

Lesson 14 (cont.)

i. Students draw a line segment through the center terminated by the circle at both ends.
ii. The line drawn in (i) is also called a diameter.
iii. An infinite number of such line segments can be drawn.

d. Every diameter is made of two radii.
e. Have students choose any point on the circle, named point M. Trace a pencil point about the circle, (on the line) in a clockwise direction until the pencil point returns to point M. the distance traveled the circumference of the circle.
f. Review construction of a circle given center and radius.

Assignment:

1. Study notes (quiz tomorrow. Students are responsible for spelling).
2. On cardboard (8" x 8") construct a circle with a 7" diameter. Draw a line segment 7" long on paper and mark it in inches. Cut out circle and bring it to class.

Lesson 15

1. Circumference of a circle.
   A. Quiz
      1. Define adjacent angles.
      2. Name a pair of adjacent angles in the figure.
      3. Name the parts of a given circle and related parts.
   B. Review parts of a circle.
      1. Stress interior and exterior radius and circumference.

Lesson 15 (cont.)

C. Refer to homework assignment. Every student expected to hold up the disk which he had cut out.

1. Classwork: teacher draws a line on board about one meter long. Ask for student to bring his disk to board.
    - a. Two students hold meter stick against line drawn.
    - b. First student places his disk on arbitrary point on line and marks point where disk touches line.
    - c. Roll disk on meter stick until point on disk has again touched line (one complete revolution).
    - d. Mark new point on line.
    - e. Remove meter stick and count number of diameters in the newly formed stretchout of the circle.
    - f. This measure is the circumference of the circle.
        - i. The unit is the diameter of the circle.
        - ii. Circumference = 3 diameters + 1/7 diameter.
2. Check to see that this works for other circles.
    - a. After more classwork, conclusion (above) should be acceptable.
    - b. Students will emphasize that   is the symbol for this factor: 3 1/7 is a usable approximation.
3. Classwork: find the circumference of each circle for which the following measures are given.
    - a. 7 radius, 28 diameter, 50 radius, 20 diameter.
4. Standard unit of measure for an angle.
    - a. The degree is the standard.
    - b. Arbitrary division of a circle into 360 equal arcs.
        - i. A small angle formed by drawing radii from each endpoint of an arc to the center is called a degree.

Name ______________________ Date ___________

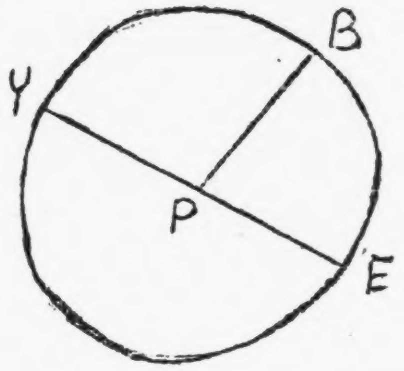
Quiz: Lesson 15

1. Define: adjacent angles

2. Name a pair of adjacent angles in the figure below. ___________

3. Name the parts in the figure:

   a) Point P is _______________
   b) $\overline{PB}$ is _______________
   c) $\overline{YZ}$ is _______________
   d) The length of the circle is called _______________



c. Have students refer to the disks which they made.

   i. Fold the circle along a diameter.
   ii. How many degrees in half a circle?

d. Introduce symbol for degree (0).
e. Open the disk again.

   i. Construct the bisector of the diameter and cut the disk along the bisector creating two semicircles.
   ii. Fold each semicircle along the original fold, creating one-quarter of a circle.
   iii. How many degrees in a quarter-circle?

Assignment:

1. Find the circumference of circles, given the following:
   a. radius = 56 miles, radius = 63 miles, diam. = 63 miles.
2. Study notes.

Lesson 16

I. The degree

   A. Review
      1. Circle, parts of a circle, and related measures.
      2. Angle measure, arbitrary units, and standard unit-the degree.

   B. Develop concept of approximate size of one degree.
      1. Using the quarter-circle constructed yesterday, repeatedly bisect the angles down to 11 1/4 degree angles.
      2. On another quarter-circle, estimate 1/90th of a quarter-circle.

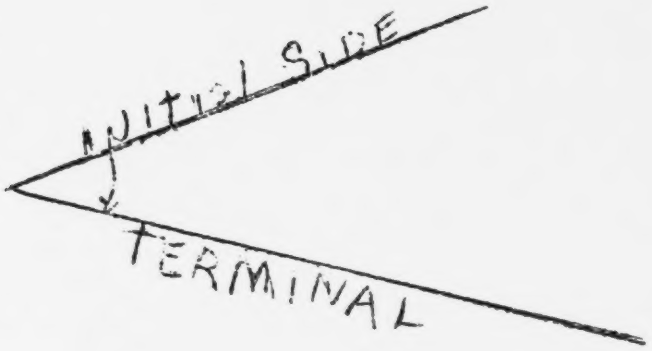
3. Student reaction: a degree is very small.
   This is not so. Discuss.
4. Demonstrate, by showing a small angle constructed in two concentric circles.
   a. Emphasize that the arc cut by the rays on the larger circle is longer than the corresponding arc on the smaller circle.
   b. Indicate that the further the rays are extended, the further apart they become. Illustrate at board.

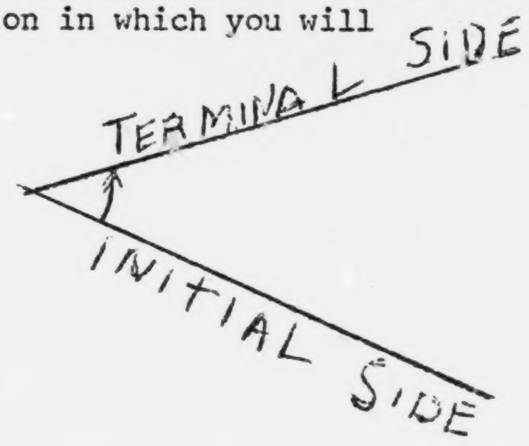
      i. Thus, a degree could mean a long distance of arc between them as we travel miles from the vertex.
      ii. This could make a big difference on a moon shot, or some related situation.

C. Introduce the protractor.
   1. Demonstrate different sizes. Use overhead projector.
      a. Show circular protractor.

         i. Show two directions of measurement of angles.
         ii. Clockwise
         iii. Counterclockwise

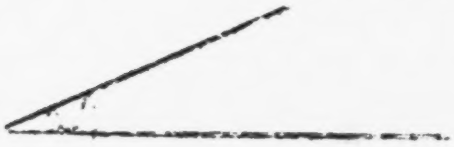
   2. Use of protractor.
      a. Draw angle, label initial side and terminal side, show direction in which you will measure.

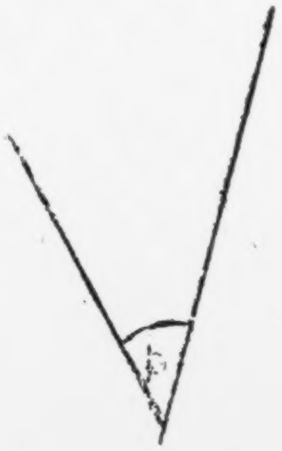


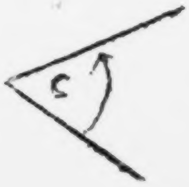


Assignment: Lesson 16

I. Label the initial side and the terminal side of each angle. Then measure each angle and record the measure.

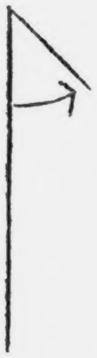


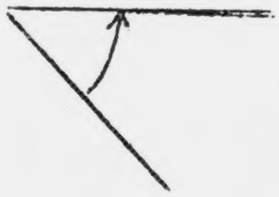


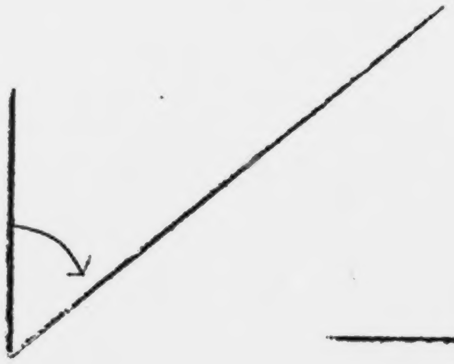


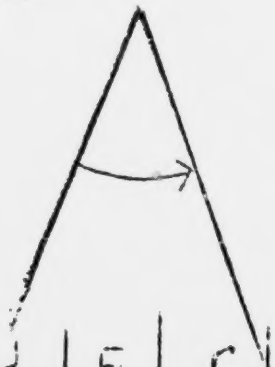
angle	a	b	c
measure			

II. Label the right side and the left side of each angle. Then measure each angle and record the measure.



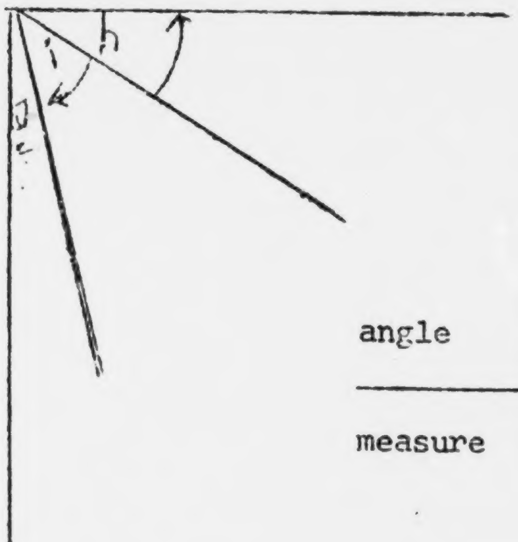


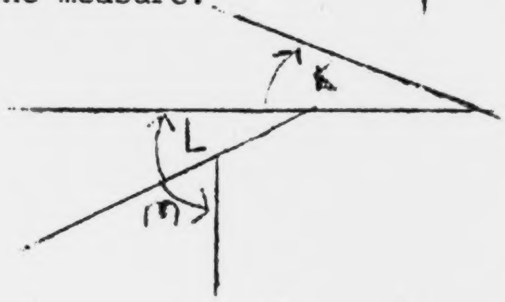




d	E	f	G

III. Measure each angle and record the measure.





angle	h	i	J	K	L	M
measure						

3.
  a. Always measure from initial side to terminal side.
  b. We may measure clockwise or counterclockwise. Show examples.
  c. We may refer to sides of angle as right side and left side.

     i. Demonstrate: Approach vertex from exterior part of name right and left side.

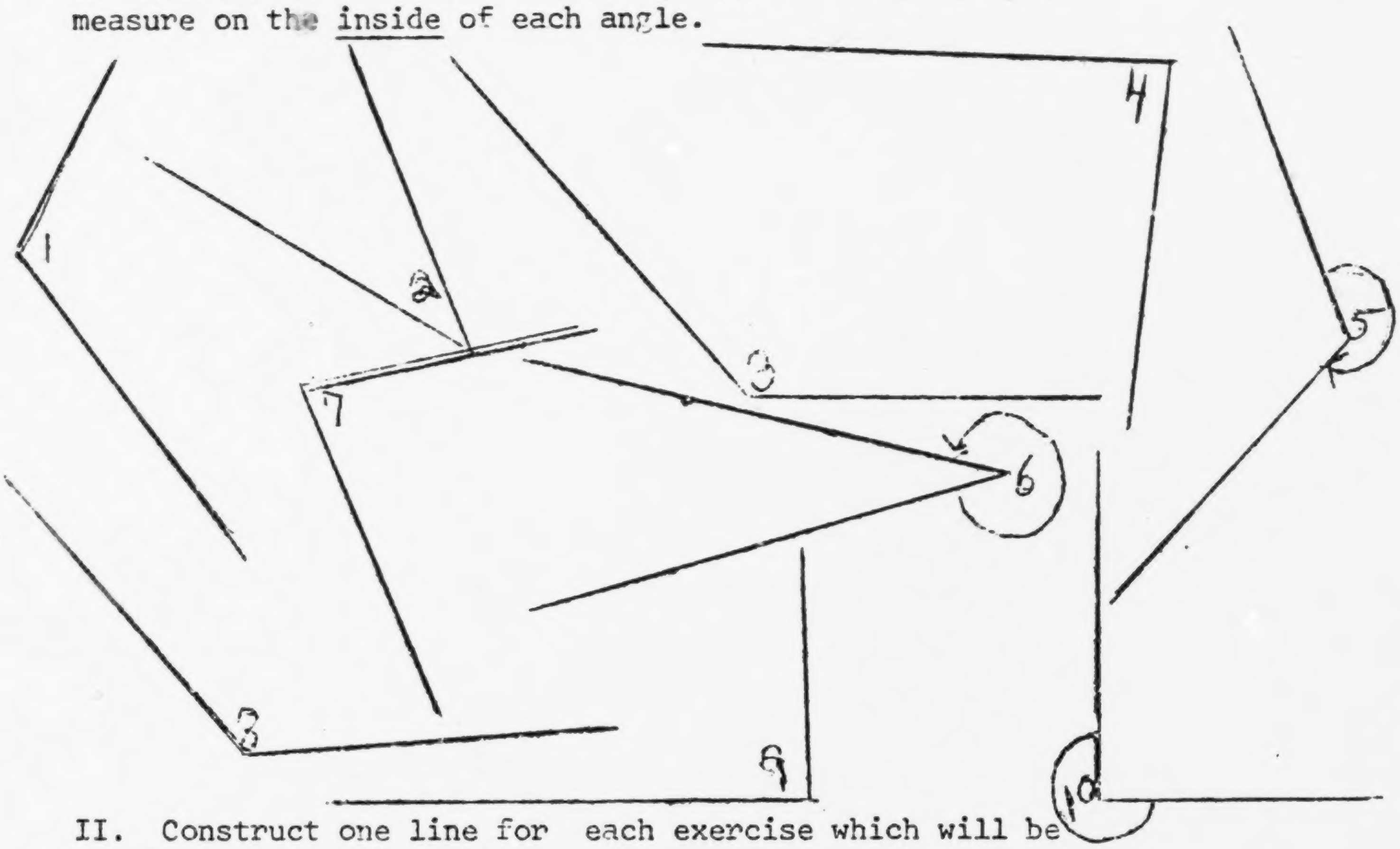
D. On hectographed page having various angles drawn. show initial side and terminal side on each angle and draw arrow to indicate direction in which to measure.
  1. Warn students that terminal side must be drawn long enough so that it will intersect the degree scale of the protractor.
  2. Name right side and left side in first five angles.

Assignment: Complete classwork assignment (part D) for homework.

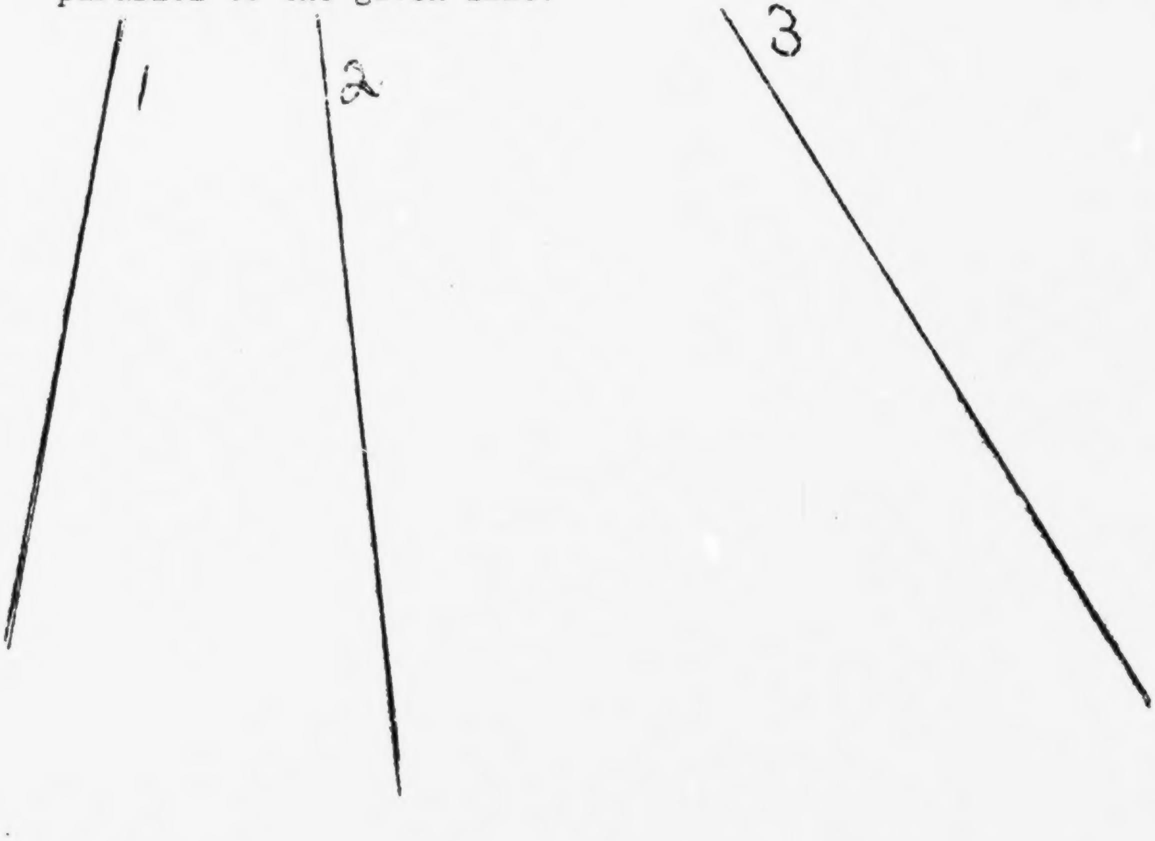
Name____________________ Lesson ______________

Assignment: Lesson 17

I. Measure each of the following angles. Indicate your measure on the inside of each angle.



II. Construct one line for each exercise which will be parallel to the given line.



Lesson 17

I. Angle measurement

  A. Review measuring angles.
    1. Use overhead projector and clear plastic protractor.
      a. Indicate two scales on protractor.

        i. Clockwise and counterclockwise.
        ii. Scale used depends upon choice of initial and terminal rays.

    2. Compare markings on a circular protractor and 180-degree protractor.

  B. Review homework.

  C. Classification of angles.
    1. Demonstrate on overhead projector with classwork.
      a. Zero degree angle.
      b. 90 degree angle - right angle.
      c. 180 degree angle - straight angle.
      d. Acute angle

        i. An angle whose measure lies between zero and 90 degrees.

      e. Obtuse angle

        i. An angle whose measure lies between ninety and 180 degrees.

      f. 360 degree angle.

  D. Measure of angles between 180 and 360 degrees.
    1. Compare methods of measuring these angles using a circular protractor and a 180 degree protractor.
    2. If such an angle is of interest, an arrow will indicate such.

  E. Parallel lines.
    1. Definition: two lines in the same plane which do not meet no matter how far they may be extended.
    2. Demonstrate construction of two parallel lines.

      i. Use the theorem that if two lines are perpendicualr to the same line, then they are parallel.

Lesson 17 (cont.)

Assignment:

1. Measure the following angles.
   (Angles given vary from zero to 360 degrees).
2. Construct lines parallel to the given lines.

Lesson 18

I. Angle measurement and terminology of triangles.

   A. Quiz on terminology of angles.

   B. Review quiz.

   C. Review of homework
      1. Measurement of angles.
      2. Construction of parallel lines.

   D. Terminology of triangles.
      1. Definition: a triangle is aaclosed plane figure formed by three line segments.
      2. Demonstrate with a figure the names of the parts of a triangle.
         a. Three sides.
         b. Three interior angles.

            i. At this time, omit exterior angles.

         c. Vertices (named with capital letters).

            i. When naming the triangle, one must always give the names of the vertices in clockwise or counterclockwise order.

         d. Sides: demonstrate names on diagram.

            i. Named as line segments with vertices as endpoints.
            ii. May be named with small letters corresponding to the names of the opposite vertices.

Lesson 18 (cont.)

- E. Property
  - 1. Demonstration to class
    - a. Draw an acute triangle.
    - b. Construct a perpendicular from any vertex to the opposite side.
    - c. Cut out the triangle.
    - d. Fold triangle so that the three vertices are concurrent with the foot of the altitude which you constructed.
    - e. Notice that the three angles fit in a straight angle.
  - 2. Other demonstrations to class.
    - a. Perform same work with obtuse and right triangles.
      - i. Draw perpendiculars from the vertex which locates the obtuse angle or right angle.
    - b. Observe same result as above.
  - 3. Conclusion: the sum of the interior angles of any triangle is 180 degrees.
  - 4. Classwork, with aid of overhead projector.
    - a. Given the measure of two angles, find the measure of the third angle of a triangle.
      - i. ∢ A = 60°, ∢ B = 50°, find angle C.
      - ii. ∢ B = 60°, ∢ A = 30°, find angle C.
      - iii. ∢ B = 45° ∢ A = 45°, find C.
      - iv. ∢ B = 20°, ∢ C is a right angle, find A.
      - v. ∢ A = 60°, ∢ C = 60°, find angle B.
      - vi. ∢ C = 25°, ∢ B = 25°, find angle A.
      - vii. If two angles of a triangle are equal and the third angle measures 80 degrees, what is the measure of one of the equal angles? How do I go about solving this problem?

Lesson 18 (cont.)

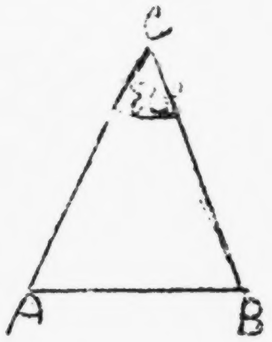
F. Methods of solving problems involving geometric concepts.
   1. Draw a picture or sketch showing the conditions given in the problem. Thus:

      i. Draw a triangle and label the parts given. Try to have given parts of the figure appear in proportionate sizes.

      ii. Example:
      Given: Triangle ABC with
      ∠ C = 80°
      ∠ A = ∠ B

      Problem: Find ∠ A = ________

      ∠ B = ________



      iii. Having listed the information, a sketch, and the desired measures, then analyze the problem.

Assignment: On hectograph pages.

1. If two angles of a triangle are equal and the third angle measures 120°, what is the measure of the other two angles?

2. If two angles of a triangle are equal and the third angle measures 30°, what is the measure of the other two angles?

3. If the angles of a triangle are related such that the second angle is twice as large as the first and the third is three times the size of the first, find the measure of each of the three angles. (Do not put too much emphasis on this problem).

Lesson 19

I. Construction of triangles using compass and straightedge.

   A. Quiz
      1. The sum of two angles of a triangle is 85°/ Jpw :arge is the third angle? Show your work.
      2. A triangle has two equal angles. The third angle measures 40°. How large is one of the equal angles? Show your work.

Name ________________ Date __________

Quiz: Lesson 18

For each problem write the correct name which will complete the statement. Credit will be given for spelling.

1. A ninety degree angle is called (a, an) _________ angle.

2. An angle whose measure is 180 degrees is called (a, an) _____________

3. An angle whose measure is between zero degrees and ninety degrees is called (a, an)________________ angle.

4. An angle whose measure is between ninety and 180 degrees is called (a, an) ______________ angle.

B. Review homework

C. Classification of triangles.

1. Scalene: a triangle in which no two sides have the same measure.
    a. Give examples.

2. Obtuse: a triangle in which one angle is an obtuse angle.

3. Acute: a triangle in which all angles are acute angles.

4. Right: a triangle in which one angle is a right angle.

5. Equilateral: a triangle in which all sides are equal.
    a. Note that all angles are also equal. Specifically 60°.

6. Isosceles triangle: a triangle in which two sides are equal.
    a. Note that the two angles opposite the equal sides are also equal in degree measure.
    b. Note special isosceles right triangle. Remind them of the special plastic triangle used in Mechanical Drawing.

7. Note special right triangle 30 - 60 - 90 degree right triangle also used in Mechanical Drawing.

D. Demonstration and classwork on construction of triangles using compass and straightedge.

1. Draw a scalene triangle on board, and reproduce the triangle using compass and straightedge as a demonstration.

2. Repeat construction of another scalene triangle with each student participating step by step.

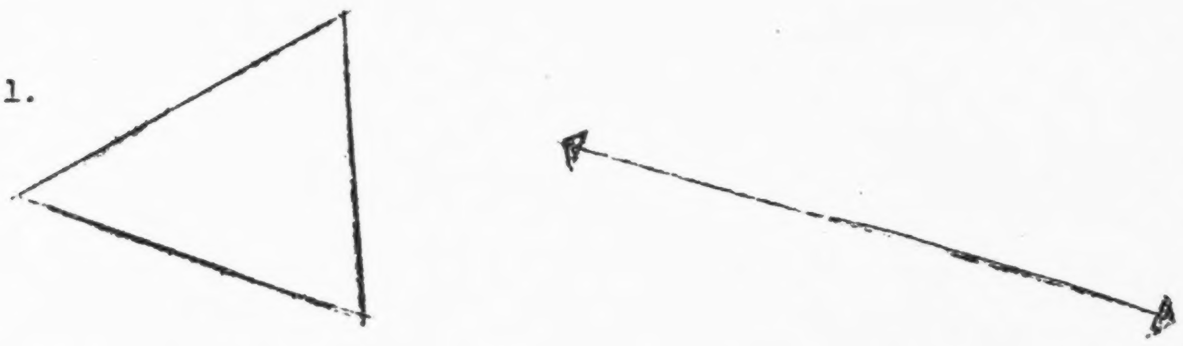
3. Repeat construction by students as classwork.
    a. Teacher helps students at their seats.

Name ______________________ Date ____________

Assignment: Lesson 19

For each exercise reproduce the given triangle using compass and straightedge. Begin construction on the given line.

1.



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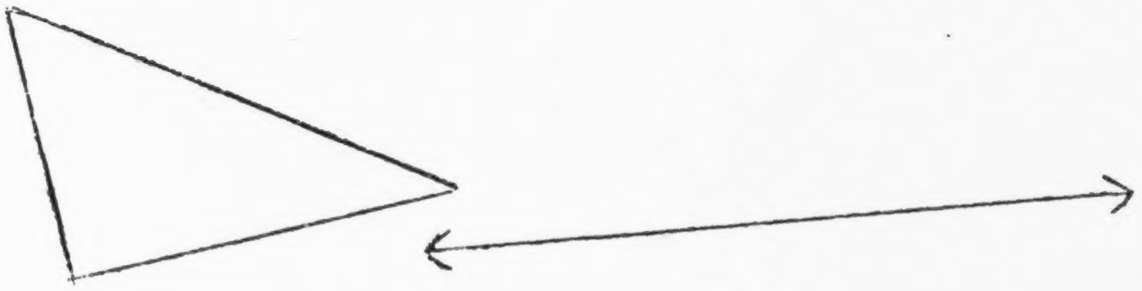

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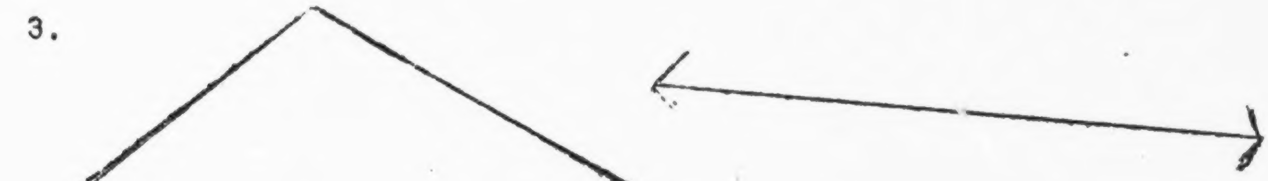

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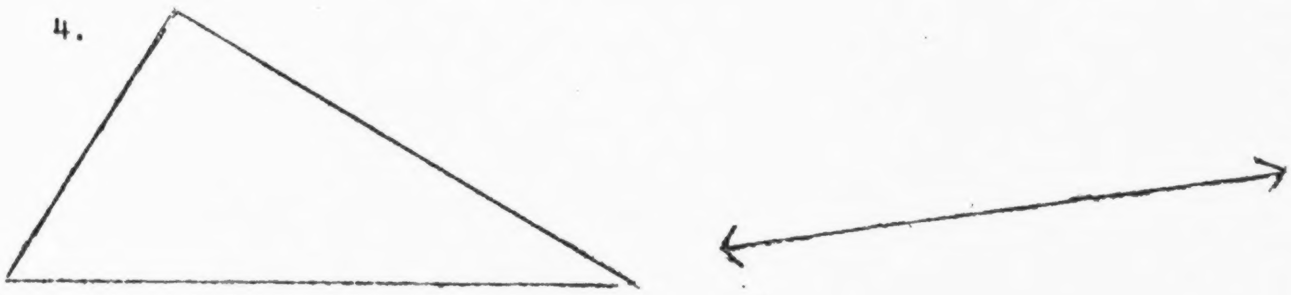


Lesson 19 (cont.)

4. Construct an equilateral triangle, given one side.
   a. Discuss, what is meant by being given one side.

      i. Since all sides are equal, we are really given 3 sides.

   b. Complete the construction as classwork.

Assignment: Study definitions and spelling (quiz tomorrow). On hectograph page, reproduce equilateral triangle, and acute triangle, an obtuse triangle, and a right triangle.

Lesson 20

I. Altitudes of a triangle

   A. Quiz: definitions of terms pertaining to types of triangles

   B. Altitudes and bases of a triangle.
      1. Identification of parts.
         a. Every triangle has three altitudes and three corresponding bases.

            i. Note general meaning of "base".

         b. Demonstrate at blackboard starting with an acute triangle.

            i. Label sides a, b, e.

            ii. Label corresponding altitudes h_a, h_b, h_c.

            iii. Explain use of subscripts.

            iv. Explain use of h representing height, or altitude.

         c. Have students practice naming sides and altitudes of triangles, using examples on overhead projector.

NameName _______________ Date _____________

Quiz: Lesson 20

1. Define: scalene triangle

2. How can I identify an obtuse triangle?

3. Define: right triangle

4. Define: acute triangle

5. A triangle with three equal sides is called (an, a) _________ triangle.

6. A triangle with two equal sides is called (an, a) __________ triangle.

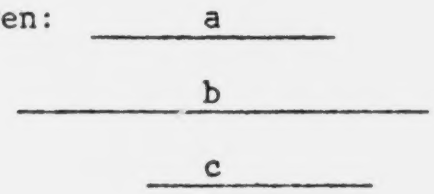
7. Define: obtuse triangle

Name ____________________ Date __________________

Assignment: Lesson 20

For each exercise construct a triangle having the parts given. Label all vertices appropriately. Then measure the angles of each triangle and find the sum of the angles.

1. Given:



2. Given: $B = 65^\circ$
$= 3.2$ cm.
$c = 4.5$ cm.

3. Given: $\underline{a} = 1\ 1/2$ inches

$\underline{b} = 5$ inches

$\underline{c} = 3$ inches

4. Given: $a = 40^\circ$

5. Given: $a = 34^\circ$
$B = 60^\circ$
$\underline{a} = 2\ 1/2$ inches

Lesson 20 (cont.)

2. Classwork
    a. Construct the three altitudes for each given triangle.
    b. Begin with obtuse triangle, in which base must be extended.

Homework: Continue work assigned in class.
Include obtuse triangles, equilateral triangle and right triangle.

Lesson 21

I. Construction of triangles.

A. Quiz: Construct all altitudes for the given triangles and label each side and each altitude.

B. Review quiz and homework
1. Does the altitude always fall inside the triangle?
2. Do the altitudes meet at a common point?
3. What is the sum of tha angles of each triangle?

C. Review construction of a triangle given three sides.
1. Scalene with acute angles.
2. Scalene with obtuse angle.
3. Equilateral given one side.
    a. In this case what is implied when one side is given?
4. Discuss: how many parts (sides) must be given so that we can construct an isosceles triangle?
5. Construct an isosceles triangle given base and one of the equal sides.

D. Review method of labeling vertices and sides of triangles.
1. Use the drawings made (above) to practice this.

E. Classwork assignment
1. Construct triangles given certain parts.

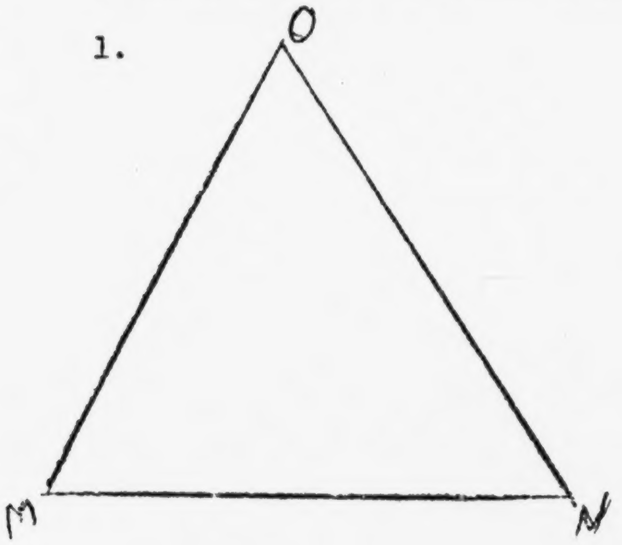
Assignment: Continue classwork and complete hectographed page.

Name ______________ Date ________________

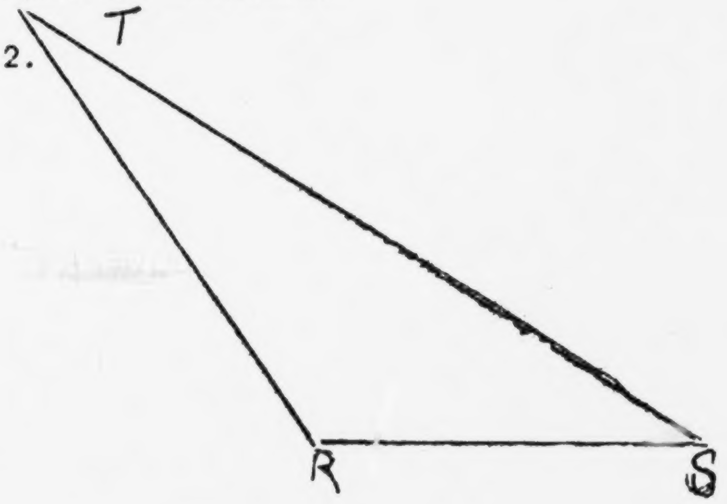
Quiz: Lesson 21

For each problem construct all altitudes for the given triangle. Then label each side and each corresponding altitude.

1.



2.



Name ______________________ Date ___________

Classwork and assignment: Lesson 21

For each exercise construct the required triangle using the given parts. Label the sides and vertices of the completed triangles.

1. Construct an isosceles right triangle given leg a. Refer to notes for parts of a right triangles. a

2. Construct an equilateral triangle given side c.

c

3. Construct an isosceles triangle given the side b and the altitude to side b.

b

h

b

4. Construct an isosceles triangle given side b as base and the sum of the two equal sides (a + c).

b

(a + c)

5. Construct a right triangle given leg a and hypotenuse c.

a

c

6. Construct triangle ABC given a, b, and c. Measure the angles of triangle ABC and classify the triangle.

a

c

Lesson 22

I. Reproducing triangles

- A. Review homework
  - 1. In problem six, what is the sum of the angles of the triangle?
- B. Quick review of reproducing triangles given three sides.
- C. Introduce method of reproducing a triangle given two sides and the included angle.
  - 1. Use two scalene triangles, one of which has been traced from the other.
    - a. Compare sizes of corresponding angles.
    - b. Measure two angles, ask for prediction of the size of the third angle.
      - i. Use intuitive concept that the sum of the angles of a triangle must be 180 degrees.
  - 2. Classwork and demonstration: given sides $\underline{b}$ and $\underline{c}$ and angle $\underline{A}$ of triangle ABC, construct a triangle with the same shape and size.
    - a. Compare the triangles using tracing paper.
    - b. Note that the two given sides must be sides of the given angle.
  - 3. Classwork: construct triangle given:

    $\angle A = 23^\circ$, $\underline{b} = 2''$, and $\underline{c} = 2\ 3/4''$.
- D. Demonstration and classwork: construct a triangle given two angles and their common side (A.S.A.)
  - 1. Given: $\angle A = 30^\circ$, $\underline{c} = 2''$, and $\angle B = 70^\circ$.
  - 2. Compare the finished triangle with the original by using tracing paper or by cutting one of the triangles out.
    - a. Label corresponding vertices using prime marks.

Lesson 22 (cont.)

E. Introduce term congruent figures and apply to the concept of congruent triangles which were constructed given certain parts of the triangle.
   1. Congruent: a term referring to the property of two figures which have exactly the same shape and size.

Assignment: Construct triangles having the parts given. Label the vertices appropriately. Then measure the angles of each triangle and find the sum of the angles. Review for test.

Lesson 23

I. Constructing triangles

   A. Quiz
      1. Construct a triangle given an angle and the sides of the angle in the triangle.
      2. Measure the size of the angles of the triangle.

   B. Refer to homework problem number 3.
      1. How many were able to construct the triangle?
         a. Try the construction on the board.
         b. Show that the sum of any two sides of a triangle must be greater than the third side.
         c. Relate this concept to practical situations.

            i. Cutting across a lot to make walking distance shorter.

   C. Review other homework: problems on board.
      1. Have triangles drawn for homework cut out and compared for congruency.

   D. Review for test
      1. Vocabulary: Isosceles triangle symbolism used to mark equal parts of triangle, definitions, etc.

Assignment: Study and review for test.

Name ____________________________ Date ______________

Quiz: Lesson 23

1. Construct a triangle having the given parts: Label the vertices appropriately.

Given: $\underline{a}$ = 3.7 cm.
$\angle \bar{C}$ = 43°
$\underline{b}$ = 4.2 cm.

2. Measure the size of each angle constructed in problem 1. Name them here.

Lesson 24

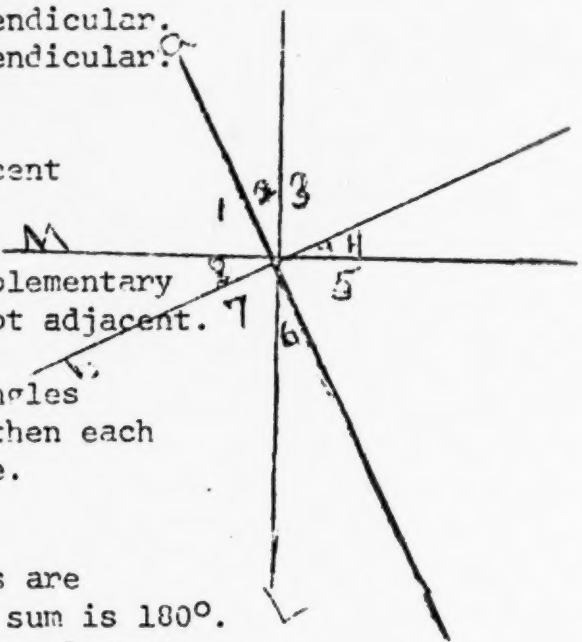
I. Test on topics to date.

Lesson 25

I. Special angles.

   A. Review test.

   B. Introduce complementary angles.
      1. Definition: two angles are complementary if their sum is ninety degrees.
      2. Give examples:
         a. Separate angles whose sum is 90°.
         b. Adjacent angles whose outer rays form a right angle.
         c. Indicate that complementary refers to a relationship between two angles.
      3. Question: what type of triangle always contains a pair of complementary angles (a right triangle).
      4. Refer to figure at right.
         Lines L and M are perpendicular.
         Lines a and b are perpendicular.
         a. Name six pairs of complementary adjacent angles.
         b. Name a pair of complementary angles which are not adjacent.
         c. Note that if two angles are complementary then each angle must be acute.

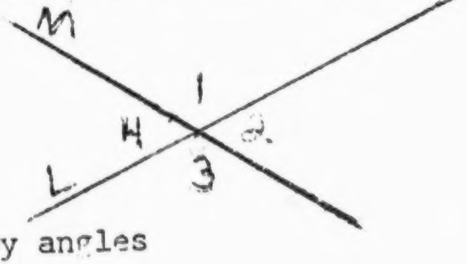


   C. Supplementary angles
      1. Definition: Two angles are supplementary if their sum is 180°.
         a. If two angles are supplementary, then one of the angles is called the supplement of the other.

Lesson 25 (cont.)

- b. Three angles whose sum is 180° are not supplementary because this term refers to a relation between only two angles.

2. Give examples:
   - a. Two separate supplementary angles.
   - b. Two adjacent supplementary angles.
3. Question: are the angles of a triangle supplementary? (no).
4. Refer to the figure at the right.
   - a. What kind of line is line L?
   - b What kind of line is line M?
   - c. Name the pairs of adjacent angles in the figure.
   - d. Name all pairs of complementary angles in the figure. (none).



5. Name all the pairs of supplementary angles.
   - a. Discuss how it is impossible for a pair of adjacent supplementary angles to be acute.
   - b. Two possible sets of angles.

     - i. Two right angles.
     - ii. One obtuse and one acute angle.

Assignment: On hectograph page, give diagram of intersecting lines (two perpendicular) and ask students to name all complementary angles and supplementary angles.

Lesson 26

I. Special angles.

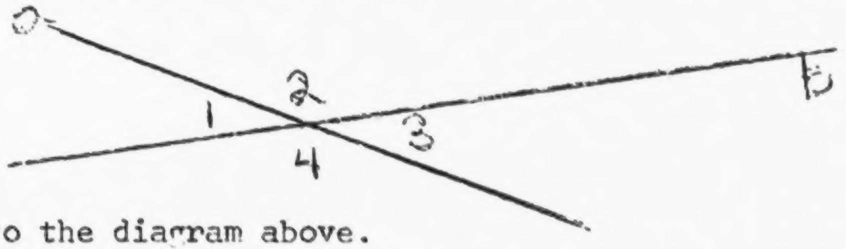
A. Quiz
1. Define complementary angles.
2. Define supplementary angles.
3. Define adjacent angles.
4. Two adjacent angles are supplementary. The acute angles measures 88° How much does the obtuse angle measure?

Lesson 26 (cont.)

B. Review quiz
   1. Discuss problem #4 in detail.
      a. Stress meaning of the term supplementary.

C. Review homework
   1. Refer to #4 and #5. Are three angles supplementary? Why?
      a. Refer to definition of supplementary.
   2. Explain #6 by assigning numerical values to the angles.

D. Vertical angles
   1. Definition: Vertical angles are two angled formed by a pair of lines in a plane and whose sides are opposite half-lines having the same vertex.
   2. Demonstrate the definition with diagrams.



E. Refer to the diagram above.
   1. What relationship do the angles 1 and 2 have?

   2. What relationship do angles 2 and 3 have?

   3. What relationship do angles 1 and 3 have?

   4. Suppose $\sphericalangle 2 = 160^{\circ}$
      a. Find the measure of $\sphericalangle 1$ and $\sphericalangle 3$.

   5. What relationship do the angles 1 and 3 have?
      a. Conclusion: vertical angles are equal.

   6. Repeat the steps 2 through 5 to develop concept again.

Assignment: On hectograph page, present diagram of intersecting lines, some at right angles, and ask students to identify relationship between certain angles.

Lesson 27

I. Construction of rectangles.

A. Review homework.

B. Review types of and relationships between angles.
1. Acute, obtuse right.
2. Adjacent, supplementary, complementary
3. Supplementary, complementary.
4. Vertical

C. Review types of triangles.
1. Scalene, obtuse, acute, right, equilateral, isosceles.
2. Special right triangles
a. Scalene: 30-60-90
b. Isosceles: 45-45-90
c. Other

D. Rectangles.
1. Definition: A rectangle is a four-sided polygon whose opposite sides are equal and whose adjacent sides are perpendicular.
2. Demonstration and classwork on construction of a rectangle.
a. Starting with convenient line construct a line perpendicular to it at a convenient point.
b. Mark off 5" base from foot of perpendicular.
c. Construct perpendicular to base at endpoint of 5 base.
d. Mark off 3 on each perpendicular from foot of perpendicular.
e. Connect endpoints on perpendiculars, label all vertices.
3. Class repeat construction with l = 4 , w = 2'.

E. Introduce property of the diagonals of a rectangle, using rectangles already drawn.

Assignment: On hectograph, have students construct rectangle and answer questions concerning it.

Lesson 28

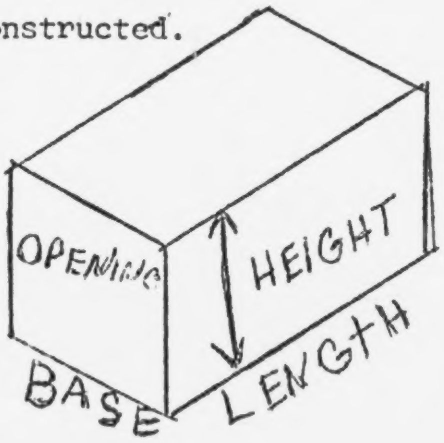
I. Construction of the stretchout for a rectangular section of duct.
A. Review homework.

Lesson 28 (cont.)

B. Diagonals of a quadrilateral.
1. Definition: Diagonal of a quadrilateral is a line segment joining vertices which are not endpoints of the same side of the quadrilateral.
   a. Illustrate with diagrams.
2. Diagonals of a rectangle.
   a. Refer to homework problems #3 and #4.
   b. Diagonals of a rectangle are equal.
3. Diagonals of a square.
   a. A square is a rectangle in which the adjacent sides are equal.
   b. Diagonals of a square are equal and perpendicular.
4. Use this property for the diagonals of rectangles to see that a rectangle has been constructed correctly.

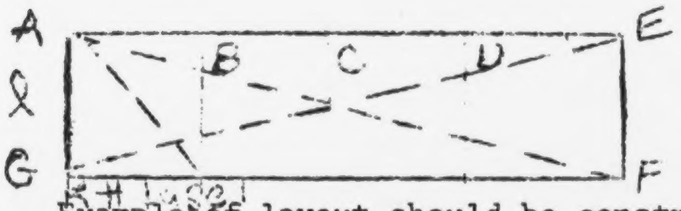
C. Rectangular duct
1. Definition: The models we shall construct are similar to the rectangular ducts used for heating and air conditioning in large buildings.
   a. Ducts are used to transport air to different parts of a building.
   b. Other types of ducts are used to control or direct water or fluids.
   c. Relate to field trip taken by members of class.
2. Properties of rectangular ducts.
   a. Parallel faces or planes.
   b. Perpendicular faces or planes.
   c. Parallel edged.
   d. Perpendicular edges
   e. Faces are a set of rectangles satisfying given measurements.

D. Construction of duct
1. Laboratory approach for construction of rectangular duct.
   a. Analysis of models previously constructed.
   b. Sketch isometric view of duct.
   c. Label dimensions
   d. Examine stretchout of model.
      i. series of rectangles having common sides.
      ii. check overall measurements
      iii. check diagonals for equality.



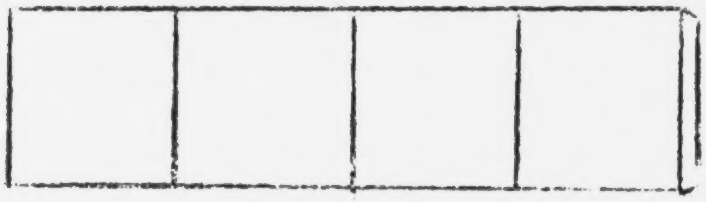
Lesson 28 (cont.)

E. Example of stretchout
   1. Check diagonal of each rectangle.
   2. Check diagonal of stretchout.
   3. Check lengths of all parts.
      a. Note equal lengths.



F. Example of layout should be constructed on blackboard as students construct at desks.
   1. Check all diagonals for equality.
   2. Check overall measurements and diagonals AF and GE for equality.
   3. Note the names given to the dimensions of the duct.
      a. Base and height are interchangeable.
      b. Length of duct is a standard notation.

G. Repeat layout of duct by class.
   1. Display model, a oblique view of board.
   2. Give and label dimensions of oblique view.
      a. Base = 3", eight = 2", lenght of duct = 4".



   3. Have class lay out duct on unlined paper.
   4. Cut out duct
   5. Teach scoring and folding of duct.

      i. Ideal to have scoring dies from paper box plant.
      ii. Use quarter to score.

Lesson 28 (cont.)

6. Discuss need for flap for gluing or stapling.
    a. Make sketch on board showing flap.

Assignment: Construct ducts given following dimensions:

1. Base = 2 1/2"	2. Base = 4 cm.
h = 1"	h = 3 cm.
l = 3"	l = 8 cm.

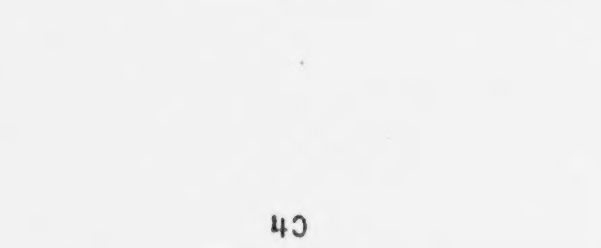
Lesson 29

I. Construction of rectangular duct.

A. Review layout of duct constructed for homework.
    1. Review notation: ase, height, length.
    2. Use of flap.

B. Cut out ducts from paper.
    1. Fold and glue.
    2. Discuss need for patience to do neat and accurate job.

C. Connecting ducts (new project).
    1. Discuss ways to connect ducts, many ways can be employed.
        a. Decide on location of flaps.
    2. Sketch layout of duct with connecting flaps.
        a. Example



D. Classwork: on cardboard layout, then cut and form a duct including connecting flaps.

Lesson 29 (cont.)

1. Dimensions: base = 6 cm., h = 4 cm., length = 10 cm.
2. Have students print their names on all fabricated ducts or layouts.

Assignment: Complete classwork.
Repeat classwork as assignment with given dimensions.

Base = 4", height = 3 1/2", length = 6".

Lesson 30

I. Connecting completed ducts: group project.

   A. Glue ducts formed for yesterday's classwork and homework.
      1. Examine ducts for neatness and accuracy in measurement
         a. Place several ducts next to each other to spot inaccuracies.
      2. Check ducts for parallel faces and perpendicularity.
         a. Does the duct lie flat on a desk when placed on any face or end?

   B. Check dimensions of connected ducts.
      1. Assign (3 or 4) students as groups and instruct them on connecting their ducts by gluing to create one long rectangular duct.
      2. Check connected ducts for dimensions and joints.
         a. Duct connections should be smooth, not in "steps" or open joints.
         b. Surfaces of connected ducts must lie on the same plane.

            i. Use flat surface to check to see that each side of the connected duct lies in a single plane.
            ii. Stand the connected duct on end to check construction.
            iii. Stand several of the ducts next to each other to check overall lengths. All should be approximately the same length.
         c. If an individual's piece is unsatisfactory, his partners are expected to help him construct a new piece in order to complete the project.

Lesson 30 (cont.)

II. Introduction to area measure.

A. Review of the development of measurement in earlier lessons.
1. Ask students to compare the length of a set of line segments.
2. Remind them of the use of an arbitrary unit of length to compare line segments.
3. Remind them of the benefits of using a standard unit of measure.

B. Draw a set of closed plane figures on the overhead projector.
1. Ask the students to compare these geometric figures by imagining placing one upon the other.
a. Indicate the difficulty of finding whether one figure is larger than another when they overlap in different places.
2. Remind students of the feasibility of using an arbitrary unit of measure.

C. The teacher presents the use of a "disk" the size of a penny for an arbitrary unit of measure for plane closed geometric surfaces.
1. For any of the figures on the overhead projector, how many pennies will fit, without overlapping each - or the edges of the figure, inside each figure?
a. Note that we cannot completely cover the inside of the figure.
b. Note that a pocket full of pennies may be necessary for demonstration.

D. Using a disk the size of a penny as a unit of measure, students are to measure the size of each of a set of plane figures. (Constructions should not create the problem of fitting parts of units).

Assignment: Complete assignment started in class dealing with step 4 above.

Lesson 31

I. Area measurement

   A. Review homework
      1. Stress problem #7 for development of concept of multiplication as a shortcut for repeated addition.
         a. This is the basis for the area formula: $A=lw$.

   B. Indicate the dissatisfaction with the coin as a unit of measure because it leaves much of the closed surface 'unused'.
      1. Discuss or demonstrate the possibility of using more of the surface as a baker would, by kneeding the unused dough, rolling it out to the same thickness and cutting more disks from it.
         a. The teacher could demonstrate this with play dough.

   C. Define area: "the number of units of a fixed shape and size which fit into a given closed plane figure".
      1. Refer to the homework assignment and tell the area of each figure.

   D. As in the unit of linear measure we can have an arbitrary unit of measure for area.
      1. The sole of a shoe.
      2. Washers stamped out of metal.
      3. Many other industrial examples which students should be able to recognize.

   E. The square as a unit of measure.
      1. Minimize the problem of using all the surface of the figure.
      2. The inch square, or square inch as a unit.
      3. The square centimeter cm. by cm. in linear measure.

   F. Discuss problem #7 from homework again and summarize the process of finding area without counting all the units:
      1. Find the number of coins which fit in one row.
      2. Find the number of rows which will fit in the figure.
      3. Find the area by repeated addition.
      4. Apply this to any rectangular figure.

   G. Classwork and assignment: Determine the number of inch squares which will fit inside each given figure without overlapping each other or the edges of the figures.

Lesson 31 (cont.)

G. Classwork and assignment:
  1. Find the number of disks the size of a penny which will fit in each rectangle. Do this without drawing all the disks.
  2. Determine the number of inch squares which will fit inside each given figure without overlapping each other or the edges of the figures.

Lesson 32

I. Area measurement

A. Quiz
  1. Define: area
  2. Find the number of inch squares which will fit in this figure. (4' x 6')
  3. Find the number of centimeter squares which will fit in the following figure.

B. Review homework
  1. Stress the process of repeated addition to shortcut the process of counting the number of units.

C. Classwork: On hectographed page. Using the unit of measure given, determine the area of each polygon. Print on graph paper having 1/2" squares. Use 1" squares as the unit of area drawn on graph paper. Stress 16 small 1/4" squares form unit.
  1. One problem involves the area of a triangle.
    a. Have students cut out the triangle. Then cut along the altitude of the isosceles triangle, put the parts together to form a rectangle, and calculate the area.

Lesson 32 (cont.)

b. A second figure will be a trapezoid as the composite of a square and a right triangle.

  i. Find area of parts separately.

c. Stress 1-1 correspondence between:
  1. length and # squares in a row.
  2. width and # rows in rectangle.

Assignment: Hectographed page printed on graph paper having 1/4" squares. Determine the number of inch squares and/or fractional parts of an inch square which will fit in each closed figure.

Use the technique of judiciously cutting up each figure and rearranging the parts to form rectilinear figures - thus making it easy to count the number of squares which will fit in each figure.

Lesson 33

I. Developing formula for the area of a triangle.

  A. Review homework
    1. Stress technique of cutting each figure in such a manner that the parts can be arranged to form rectilinear figures.

  B. Area of rectangle represented by a formula: develop formula for area of a triangle.
    1. Draw the diagonal of a rectangle and observe the creation of two right triangles.
      a. The area of each triangle is half that of the triangle.
      b. Give particular examples and determine the area of both rectangle and resulting triangle.

  C. Given a parallelogram, find its area based upon the area of a rectangle.
    1. Draw a line from one vertex perpendicular to and cutting the opposite base. Thus a right triangle is formed.

Lesson 33 (cont.)

2. Cut off this right triangle and place it next to the other side of the parallelogram so that the hypotenuse of the right triangle lies on the other side of the parallelogram forming a rectangle.
    a. The area of the rectangle can be determined from the length of the parallelogram and its height.

        i. Thus, A=bh for parallelogram.
        ii. Stress 1-1 correspondence length to # squares per row width to # rows.

D. Classwork:
    1. On hectograph page, give the basic formulas for area and have students find the area by calculation of several polygons. Show your calculations.

Lesson 34

I. Area of basic geometric figures.

    A. Review homework
        1. Students should indicate the process of finding areas of compound figures.
        2. Review calculations.

    B. Finding the length of a rectangle when given the area and the width of the rectangle.
        1. Example: Stress 1-1 correspondence of linear measures to # square units.
        2. Given: A = 96 sq. inches, L = 8"

            a. Use the 8" to tell the number of squares per row.
            b. Sketch a rectangle with length of 8".
            c. Using your knowledge of division, find the number of rows needed in the rectangle to have an area of 96 squares.

Lesson 34 (cont.)

i. $\frac{96 \text{ (number of squares contained by figure)}}{8 \text{ (number of squares per row)}}$

ii. Quotient (12) is the number of rows needed.

2. Thus: $96 = 8w$
   a. Replace 96 by 8 x 12: $8 \times 12 = 8w$
   b. Therefore: $12 = w$

C. Introduce basic axiom of division for solution of linear equations.
   1. Example:

$$96 = 8w$$
$$\frac{96}{8} = \frac{8w}{8}$$
$$12 = w$$

D. Classwork in practicing and applying the axiom for division.

Assignment: Complete classwork assignment. Practice finding missing dimension by using division axiom.

Lesson 35

I. Cross sectional area of ducts.

A. Quiz
   1. Find the area of a rectangle 10 cm. by 80 mm.
   2. Find the area of a triangle whose base is 14 inches and whose altitude to the 14 inch base is 9 inches.
   3. If the area of a triangle is 64 sq. cm., and its altitude to side a is 8 cm., find side a.

B. Review homework
   1. Stress different approaches to problem #7.
      a. Find the area of each separate rectangle and add.
      b. Find overall dimensions and determine area of large rectangle.
      c. Stress 1-1 correspondence between:

         i. length and # squares per row.
         ii. width and # rows.

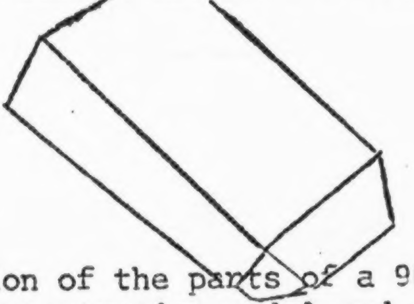
Lesson 35 (cont.)

C. Cross sectional area
   1. Definition by demonstration
      a. Imagine cutting a duct by a saw whose surface is perpendicular to the edges of the duct.
      b. The ends of the two pieces form rectangles satisfying the Base and altitude of the given duct. The area of this rectangle is the cross sectional area of the duct.

         i. Give other examples of cross sections by cutting a 2" x 4" board with a saw.

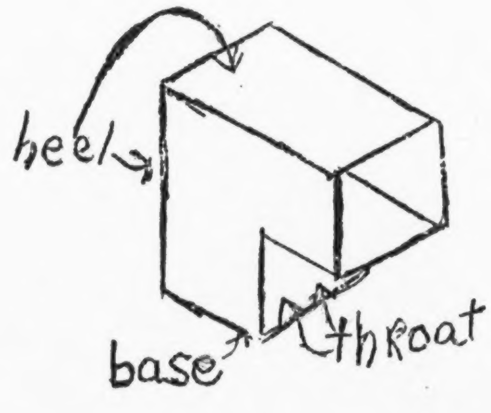
D. Classwork: Find the cross sectional area and the lateral (surface) area of a rectangular duct with dimensions of 12 cm. x 8 cm. x 25 cm.

   1. Note: dimensions are given in the order: base x altitude x length.



E. Construction of the parts of a 90 degree elbow.
   1. Use demonstration model and parts from another model.
   2. Purposes of an elbow in duct work.
      a. To direct the flow of air in a different direction.
   3. Terminology
      a. Four parts to an elbow.

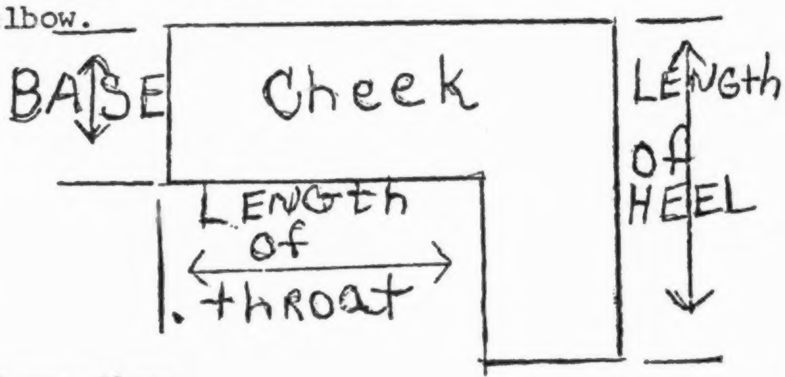
         i. Two cheeks
         ii. A throat
         iii. A heel



Lesson 35 (cont.)

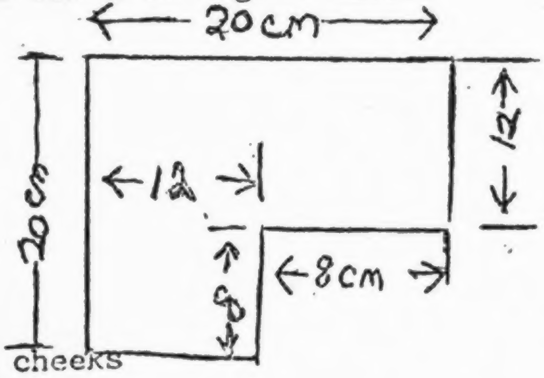
b. A flat elbow is an elbow for which the ends of the cheek act as the base of the duct, i.e., one edge of the cross section of the duct.

4. Dimensions of the parts.
   a. The length of the heel equals the sum of the length of the throat plus the base of the duct.
   b. The length of the throat determines the length of the elbow.

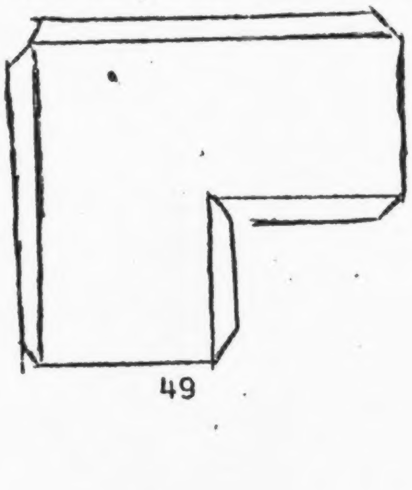


5. Forming the parts of an elbow.
   a. Form the heel in one piece.
   b. Form the throat in one piece.

F. Classwork: construct the parts for a 90 degree elbow.
   1. Dimensions: base 12 cm.
      height 6 cm.
      throat 8 cm.
   2. Construct the cheeks.

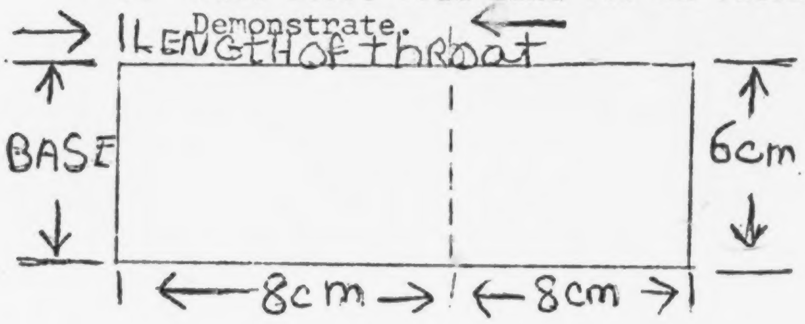


   3. Construct the flaps on the cheeks

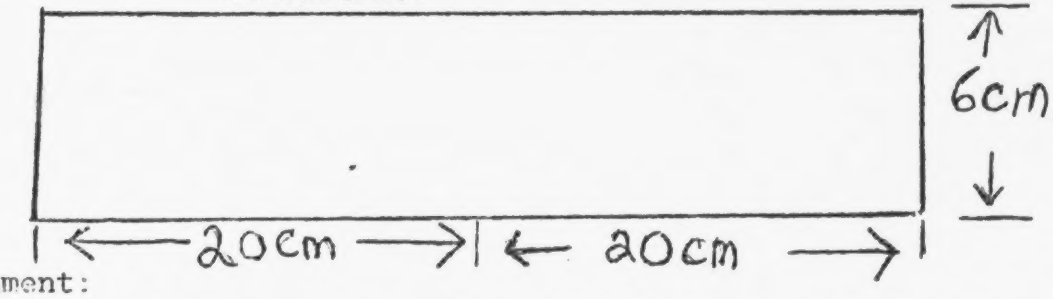


Lesson 35 (cont.)

4. The pattern for the throat is one long rectangle with a total length of two inside edges of the cheek. No flaps needed.
    a. Throat in this example is (8 + 8) cm. by 16 cm.
    b. Must score fold line for an outside fold. Demonstrate.



5. Pattern for heel is one long rectangle with total length of two outside edges of the cheek.
    a. Heel in this example is (20 + 20) cm. by 6 cm.
    b. Must score fold line for an inside fold. Demonstrate



Assignment:

1. Construct and cut out all parts for a 90 degree elbow to fit a duct whose cross section is 10 cm. x 5 cm. Include flaps on the cheeks.
2. Calculate the total surface area of the parts.
3. Be prepared to glue the parts together in class tomorrow.

Lesson 36

I. Construction of 90 degree and 45 degree elbows.

   A. Review layout of 90 degree elbow

   B. Demonstrate technique of paper folding to form the pieces for fabrication of elbow.
      1. Students report areas of each part.
         a. Check throat elbow.
         b. Total surface area.
         c. Cross sectional area
      2. Students glue parts together.

   C. Discuss cost of fabricating straight duct in comparison to elbow.
      1. Material wasted in constructing elbow.
      2. Labor time required for elbow is greater.
      3. Fittings, such as elbows require more knowledge
         a. The better the mechanic, the better his pay.

   D. Repeat the layout technique for a 45 degree elbow.
      1. The use of T-square, 30-60-90 degree and 45-right triangles should be used for these constructions.

   E. Need for connecting flaps on ends of ducts
      1. Include connecting flaps on fittings, same as on straight duct.

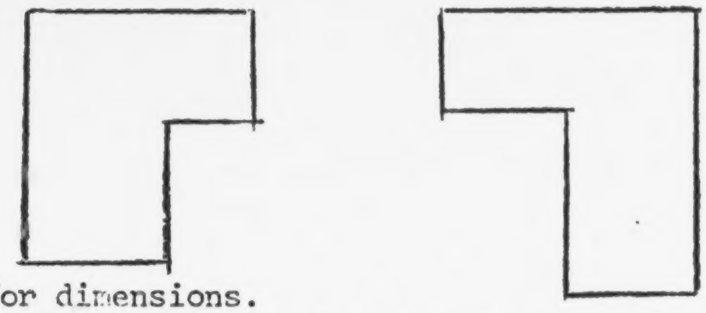
Assignment:
1. Layout the parts for a 90 degree elbow with connecting flaps. Base 4"
   Height 3"
   Throat 2
2. Cut out the parts.
3. Layout a 45 degree elbow with connecting flaps
   Base 4   height 3", throat 2"
4. Cut out the parts.
5. Determine the total surface area of each fitting.
6. Determine the cross sectional area of each.

Lesson 37

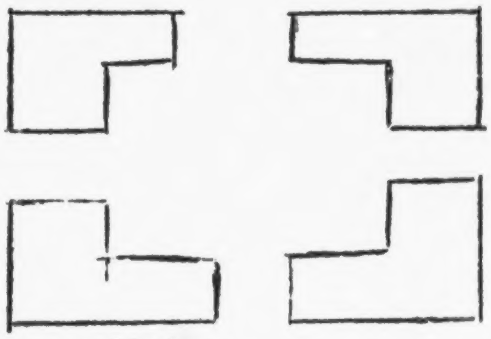
I. Elbows

  A. Form and construct homework project
    1. Check for neatness of work, tolerances.

  B. Pair up students and have them connect their elbows
    1. Example:



    2. Check for dimensions.
      a. Connected elbows should lie flat on table top, when placed on cheeks.
      b. Stand ducts on bases. The bases should lie flat.
      c. Lie ducts on top heels. Surface should lie flat.
    3. Pair four students to connect their elbows to form a complete square loop of duct.
      a. Example:

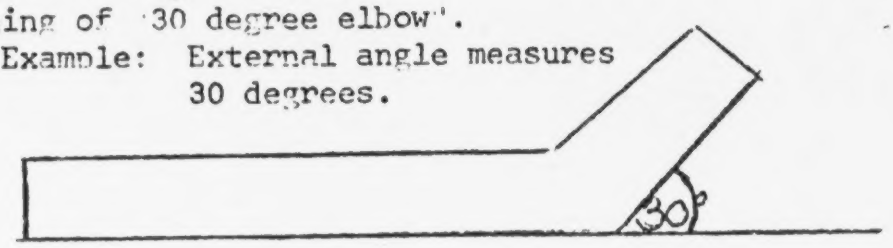


      b. Discuss good and poor conditions

        i. Point out properties which bring success.
        ii. Point out shortcomings in measurement for projects which do not fit correctly.
    4. Check 45 degree elbows
      a. Check for flat surfaces.
      b. Check for dimensions by stacking one next to the other.
      c. Glue two together to see if heels at extremities form perpendicular surfaces.

Lesson 37 (cont.)

C. Construct a 30 degree elbow.
   1. Meaning of "30 degree elbow".
      a. Example: External angle measures 30 degrees.



      b. Demonstrate construction of elbow using T-square and triangles.

Assignment:

1. Construct a 30 degree elbow: 4 parts with flaps on cheeks. Cut out and fold.
   Base: 8 cm., Height: 6 cm., Throat: 4 cm.
2. Construct a 60 degree elbow using the same dimensions.

Lesson 38

I. Elbows for duct work

   A. Fabricate last night's project
      1. Check for dimensions and angles.
      2. Check for flatness.

   B. Review method of layout for elbows.

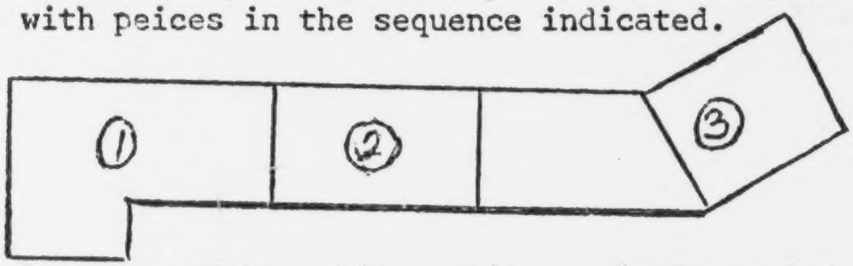
   C. Classwork and assignment.
      1. Fabricate a 90 degree elbow. Base 9 cm., height 4 cm., throat 3 cm.
      2. Fabricate a duct 10 cm. long to fit the 90 degree elbow.
      3. Fabricate a 60 degree elbow to fit the 10 cm. duct, same dimensions as the 90 degree elbow.
      4. Layout, cut out, and fold all parts.
      5. Find the total surface area of the 3 pieces.

Lesson 39

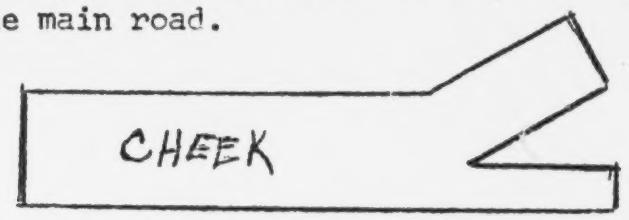
I. Branching ducts

A. Check total surface area for last night's assignments.

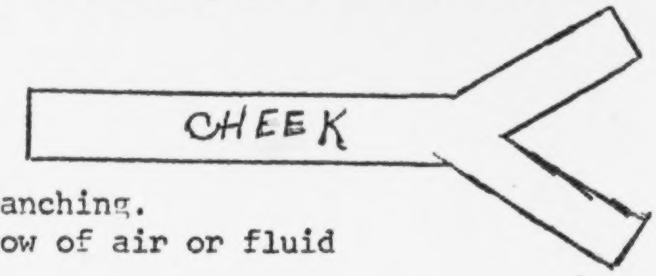
B. Fabricate (put together) pieces according to directions with peices in the sequence indicated.



C. Introduce Y-branching, using a display model.
  1. Example: concept is very similar to a fork in a road.
    a. Straight road branches to right or left from the main road.



    b. Straight road branches in both directions.



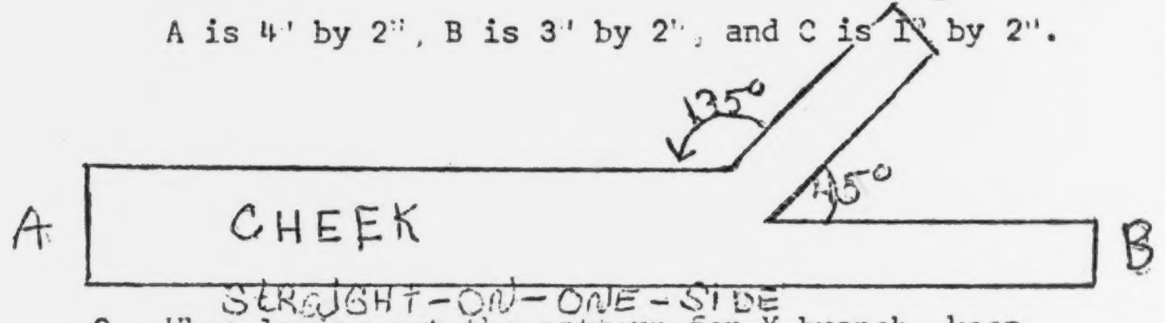
  2. Purpose for branching.
    a. Control flow of air or fluid

D. Application of measurement to Y-branching to duct work.
  1. Total cross sectional area of branches must equal the cross sectional area of the trunk (main) line.
    a. The branches are called "take offs".
    b. Reason for maintaining area: to maintain a constant pressure and constand flow of air through all ducts.

Lesson 39 (cont.)

E. Classwork with demonstration
   1. Layout Y-branch given these cross sectional dimensions and angle of branching:
      Cross sectional dimensions:

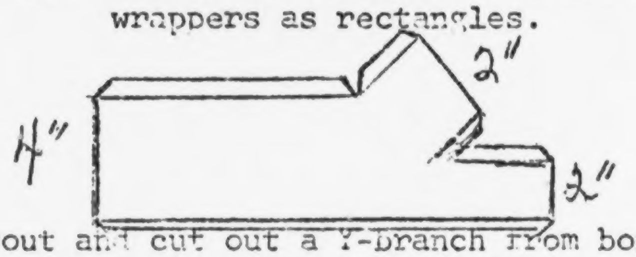
      A is 4" by 2", B is 3" by 2", and C is 1" by 2".



   2. When laying out the pattern for Y-branch, keep sides of take-off parallel.
      a. Use knowledge of elbows-construction.
      b. Supplementary angles, indicated on the drawing, are used to keep sides of branch parallel to each other.
      c. Note that for the given dimensions the areas of B and C add up to the cross sectional area of A.
      d. Tools of construction: T-square, right triangles, straight edge, ruler, protractor (not needed here).

F. Classwork: Layout a Y-branch to specifications given above.
   a. Students follow instructor's demonstration.

      i. Design a right and left cheek with flaps which fold in opposite directions.
      ii. Design stretchout of straps or wrappers as rectangles.

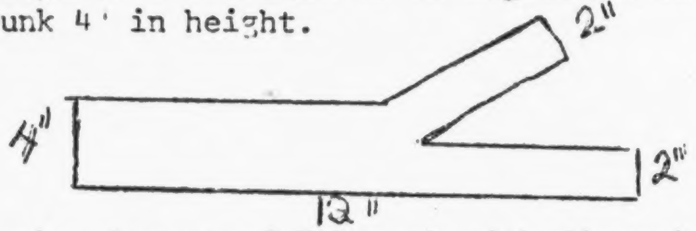


Assignment: Layout and cut out a Y-branch from boxboard to specifications given on hectograph. Three straps are in the form of rectangles.

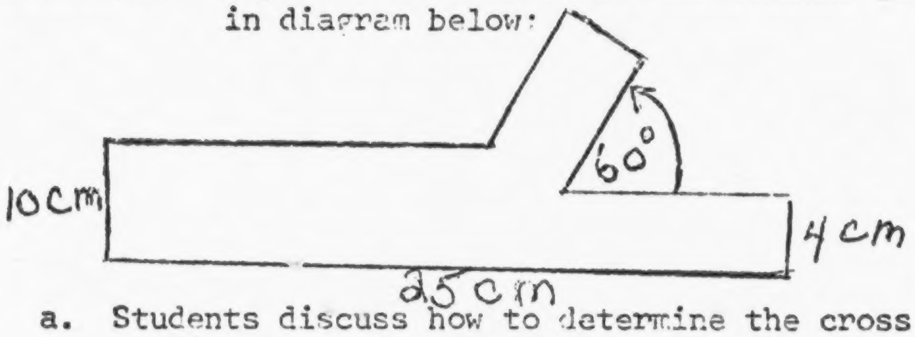
Lesson 40

I. Designing a Y-branch with attention to cross sectional area.

- A. Discuss homework and collect patterns.
- B. Review layout of Y-branch.
  - 1. Students talk the instructor through the design of a Y-branch of 30 degrees with trunk 4" in height.

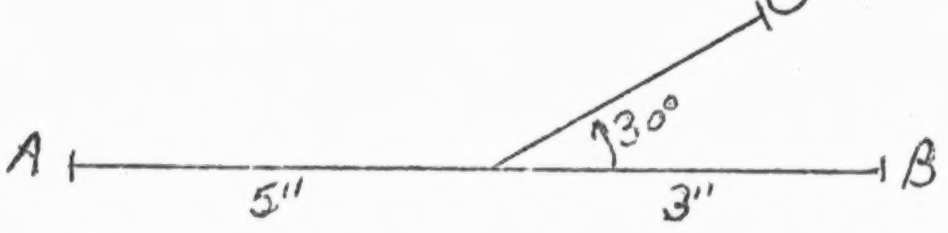


- C. Classwork: Layout of Y-branch with dimensions given in diagram below:



  - a. Students discuss how to determine the cross sectional area of the 60 degree take-off.
    - i. Using subtraction of areas, or of heights.
  - b. If time allows, start assignment.

Assignment: Layout and cut out from box board a Y-branch and straps including connecting flaps according to schematic below:
A is 5" x 2" B is 2" x 2"

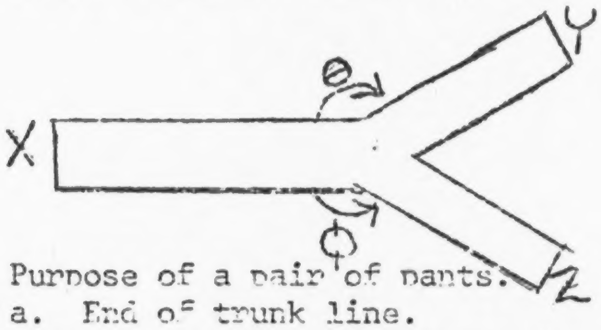


Lesson 41

I. Designing Y-branches for cross sectional area.

A. Complete homework assignment
   1. Assemble Y-branch
   2. Check for correct construction and correct measurement by comparing Y-branches.

B. Special Y-branch called a "pair of pants'.
   1. Demonstration model of the branch.



   2. Purpose of a pair of pants:
      a. End of trunk line.
   3. Discuss the different forms possible in a pair of pants.

      a. The cross sectional areas of the branches can be equal.
      b. The cross sectional areas may be unequal.
      c. The angles of the branching may be equal. θ and
      d. The angles of branching may be unequal.
      e. Combinations of the four possibilities can be used.
      f. Note that the sum of the cross sectional areas of the branches must equal that of the trunk line.

C. Demonstration of layout of a pair of pants where the cross sectional areas of the branches are equal and the angles of the branches are equal.

1. Draw a center line for the main trunk line.

2. Mark a convenient point on the center line, call it A.

3. Construct a perpendicular to the center line at A.
   a. Use T-square and right triangle.

Lesson 41 (cont.)

4. Mark off 1/2 of the base of the duct on either side of A on the perpendicular. Label these points B and C.

5. Draw lines parallel to the center line through points B and C.

6. Mark off desired length of main trunk from B and C. Label these endpoints D and E.

7. Determine the size of the angle of branching. Layout this angle at points E and D using BE as one side of one angle and CD as one side of the second angle. The angles are equal.

8. Determine the length of second side of angles, mark off distances from vertices and make new endpoints F and G.

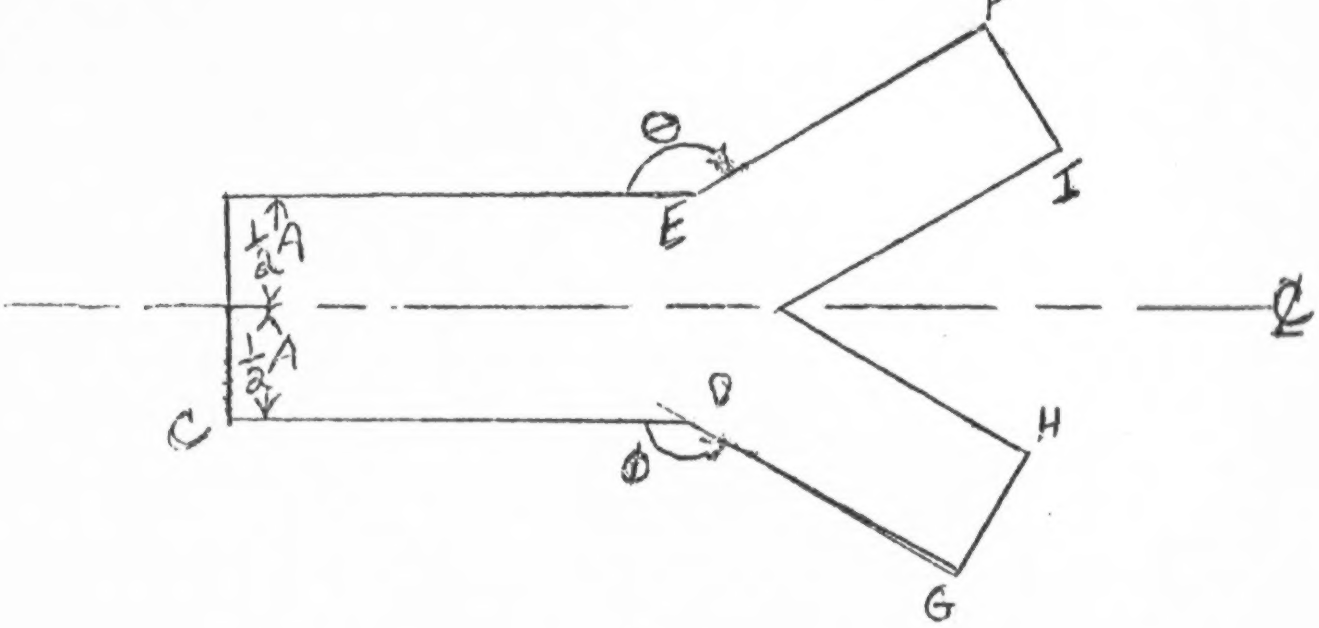
9. Draw perpendiculars to lines EF and DG through points F and G towards center lines.

10. Mark off opening of ducts along these perpendiculars such that openings will each be 1/2 the width of the main trunk. Label openings GH and FI.

11. Draw line parallel to EF through point I.

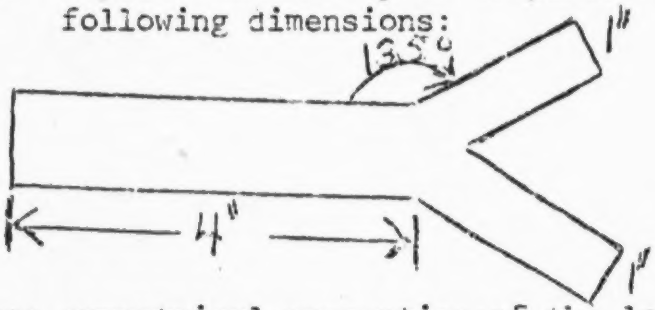
12. Draw line parallel to DG through point H.

13. Note that point K is concurrent with a point of the center line of the main trunk.



Lesson 41 (cont.)

D. Classwork: Layout Y-branch (pair of pants) given the following dimensions:



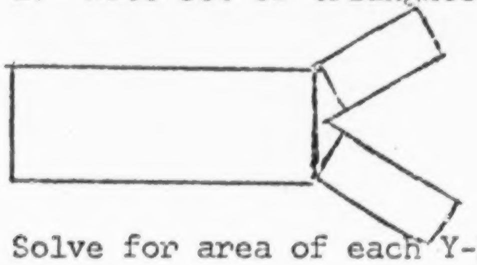
1. Discuss symmetrical properties of the layout.

Assignment: Layout and cut out three Y-branches, including straps from boxboard, as shown on hectograph.

Lesson 42

I. Designing a Y-branch

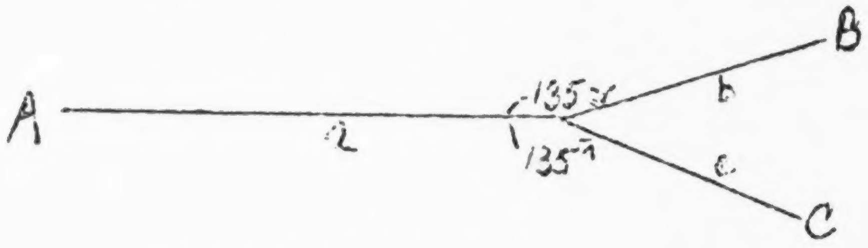
A. Fabricate Y-branches constructed for homework.
   1. Entire period spent on assembly and problem-solving involving construction and layout of Y-branch.
   2. Determine the area of cheeks of a Y-branch.
      a. Note set of triangles at intersections.



   3. Solve for area of each Y-branch given for homework.

Lesson 42 (cont.)

Assignment: Complete work on last night's assignment and layout and cut out following Y-branch, determine its area (surface).
A is 4 cm. x 3 cm. Area at B = Area at C



Include connecting flaps on end of openings.

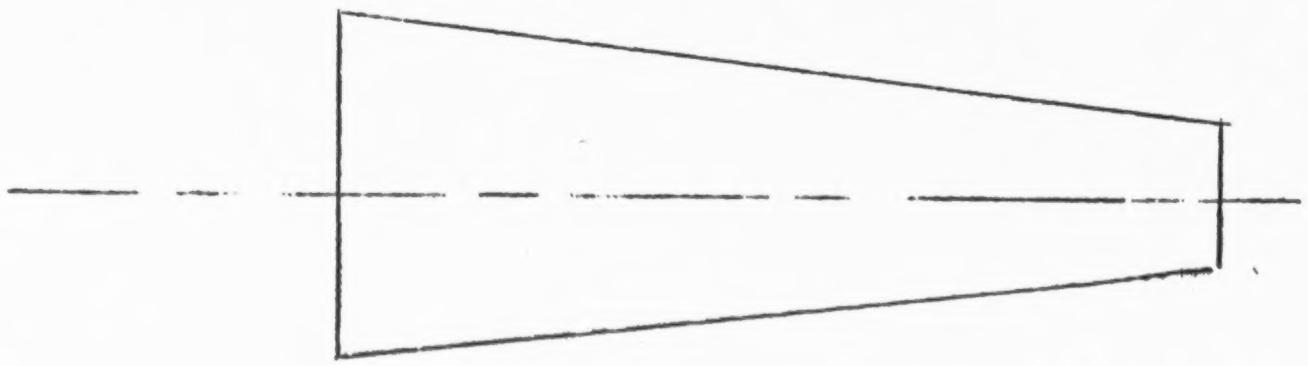
Lesson 43

I. Design of reducer

A. Complete last night's assignment
   1. Discuss expectation for completion of all assignments on time. After school time expectated if projects are not completed.

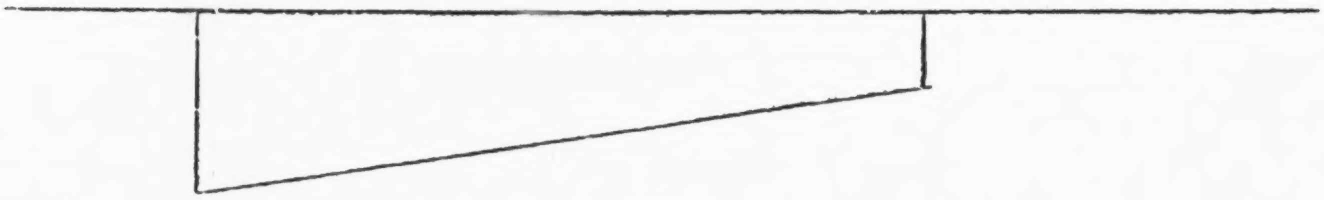
B. Reducers (transition pieces).
   1. Examples: nozzle for a hose, end of an eye dropper.
   2. Purposes: to change pressure, to conserve material, others.

C. Teacher displays models of reducers. Types:
   1. On center:



Lesson 43 (cont.)

2. Straight-on-one side:

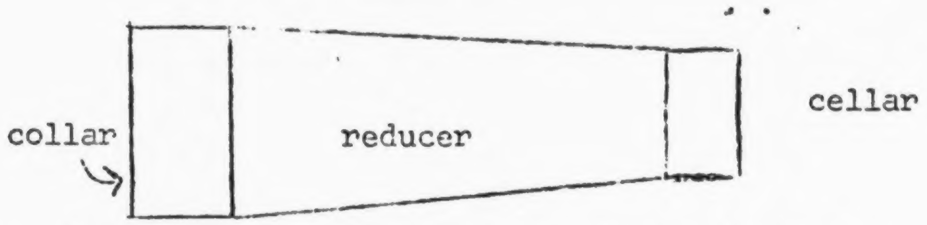


D. Demonstrate layout of reducer
   1. On center
   2. Straight-on-one side.

E. Classwork: Layout patterns for reducers of each type.
   1. On center: One end 4 1/4" by 2", other end 2 1/2" by 1", length 6".

   2. Straight-on-one side: One end 3 1/4" by 2 1/4", other end 3 3/4" by 1/2", length 5".

F. Collars on reducers.
   1. Purpose
      a. To make connections neater - having no gaps.
   2. Example:



Classwork: Construct patterns for each of the following reducers with collars. (On hectographed page).
   1. On center
   2. Flat-on-one side.
   3. Compare the two reducers.
      a. Discuss how each can be used.

         i. On center, running down center of room.
         ii. Flat-on-one side, hugging wall.

Lesson 43 (cont.)

Assignment: Layout and cut out the following reducers. Find the lateral area and cross sectional area of each end.

1. On center:

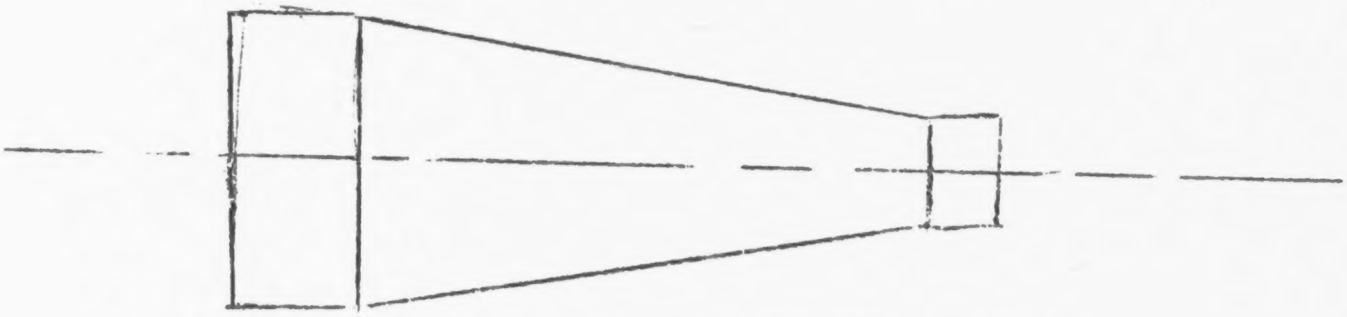
   Collars 1" long
   Distance between collars 5"
   One end 3 1/2 by 3" (large end)
   Other end 2 1/2" by 2" (small end)

2. Flat-on-one side
   Same dimensions as 1.

Lesson 44

I. Areas of the surfaces of reducers

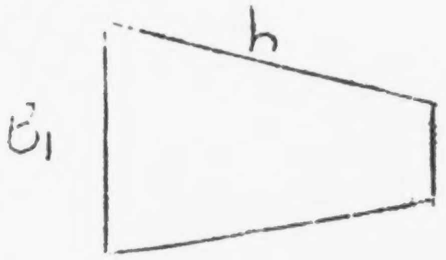
   A. Complete reducers assigned for homework.
      1. Review methods for finding area.
         a. By separating surface into rectangles and triangles.

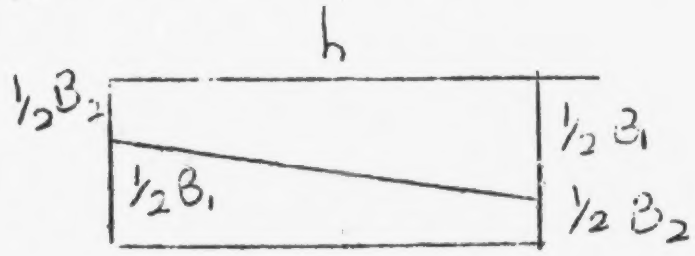


         b. By developing the formula for the area of a trapezoid.

   B. Trapezoid: A quadrilateral having two sides parallel and two sides not parallel.
      1. Develop formula for the area of an isosceles trapezoid by bisecting the figure with an altitude and reassembling the parts to form a rectangle.

Lesson 44 (cont.)





a. Area of trapezoid = $\frac{h(B1 + B2)}{2}$

2. Area formula can be tested for a trapezoid shaped like the flat-on-one-side reducer.

C. Classwork: Find the area of each of the following reducers:

1. On center: collars: 3 cm.
   distance between collars: [illegible] cm.
   large end: 5 cm. by 4 cm.
   small end: [illegible] cm. by 4 cm.
   a. Make a sketch of the reducer.
   b. Require two cheeks and two straps.
      i. Look for shortcuts in finding areas.

2. Flat-on-one side: collars: [illegible]"
   distance between collars: 8"
   larger end. 5" by 3".
   small end: 4" by 3".
   a. Note: cheeks are congruent, but straps are not.

Assignment: Hectographed sheet: Find the total area of each of the given duct systems.

Lesson 45 through 48.

I. Group projects in developing systems of duct work for display.

   A. Assign groups of class members projects in designing and fabricating models of systems of duct work.

      1. Refer to the Mechanical Drawing project.
      2. Each student is expected to contribute a part to a system of duct work.

      3. The duct system may be complicated, yet each student can contribute according to his ability.

      4. Encourage originality in the design of patterns.

UNIT 2

GEOMETRIC SOLIDS

Unit II Geometric Solids

This topic extends the use of linear measurement and precision of measurement while offering students further opportunity to experience the relation between linear, area, and volume units of measure. A great deal of manual work as well as calculations are required as part of this topic of study.

First, the teacher develops the concept of indirect linear measurement through the use of the Pythagorean theorem. This employs the knowledge of area units of measure to determine a related linear distance. The students learn to use a table of squares and square roots in order to facilitate their efforts. This knowledge is then employed to design and construct numerous patterns for a variety of geometric solids. Area formulas are developed and used.

The concept of volume is then developed as the process of counting. The teacher uses dramatizations to illustrate the concept. He has students count the number of marbles which fit in a tea cup, the number of golf balls which fit in a bucket, or the number of mathematics textbooks which fit in a carton.

The teacher next introduces a standard unit of volume: the cube. Using this unit, the students count the number of cubes which fit in a right rectangular prisms. From this stage they learn a shortcut for counting cubes. The area of the base corresponds to the number of cubes which fit in a layer. Thus a one-to-one correspondence is made between area units and volume units. The height of the solid indicates the number of layers of cubes which can be placed in the solid. The students develop the basic volume formula: V = (area of base) x height. Calculations are repeatedly made for surface area and volumeof a variety of solids.

The final project of this unit is the fabrication of a set of geometric solids from folding paper board, which is supplied by local paper manufacturers. A small group of students will decorate the Christmas tree of a local business firm with these ornaments.

The concept of volume will be employed throughout much of the remainder of the course. The topic is basic for the study of densities of metals and, later, in the study of measurement as applied to paper box fabrication and design. The concepts of area and volume are used in a wide variety of skilled trades.

Lesson 1

I. Introduction to the Pythagorean theorem.

A. Brief review of the squares of numbers.
1. Definition: the area measure of a square having given side measures.
2. Alternate definition: the number resulting from multiplying a given number by itself.
a. Examples of perfect squares.

B. Introduction to the concept of square roots of integers.
1. Definition: the measure of the side of a square having a given area.
2. Alternate definition: the number whose square is a given number.
a. Examples of square roots of integers.
b. Estimates of square roots of integers.

C. Introduction to the use of the tables of squares and square roots in the textbook.
1. Show the use of the tables and practice with them.
2. Show alternate use of the tables of squares to find or approximate the square root of a given number.
3. Show alternate use of the square root tables to find or approximate the squares of certain numbers.

D. Intorduce the meaning of the statement: $c^2 = a^2 + b^2$
1. Indicate the order of operations, meaning of exponent.

E. Review the method of labeling of parts of a right triangle.
1. Hypotenuse: the side opposite the right angle.
2. Legs or sides: the sides forming the right angle.
3. Use of capital letters to name vertices, small letters to name sides.

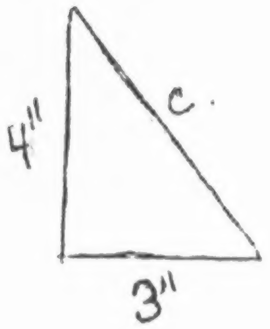
F. Teacher develops the meaning of the Pythagorean theorem.
1. Teacher demonstrates, students construct a right triangle with squares on its sides using graph paper.

Name ______________________ Date ______________________

Assignment Lesson 1

I. Find the measures of all sides of each right triangle.

1.



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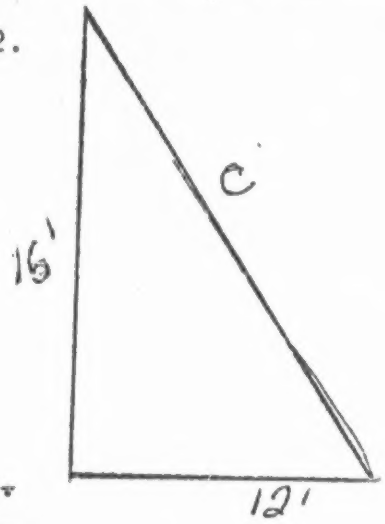

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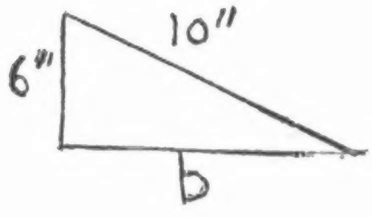

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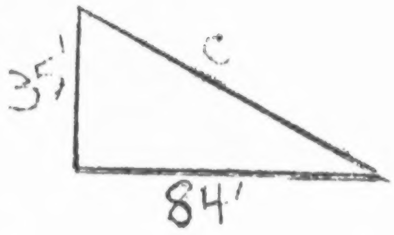

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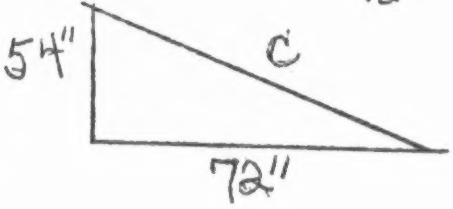

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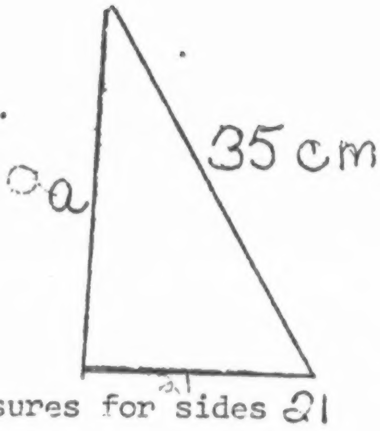


II. Is each of the following sets of measures the measures for sides of a right triangle?

7. 7", 10", 12"

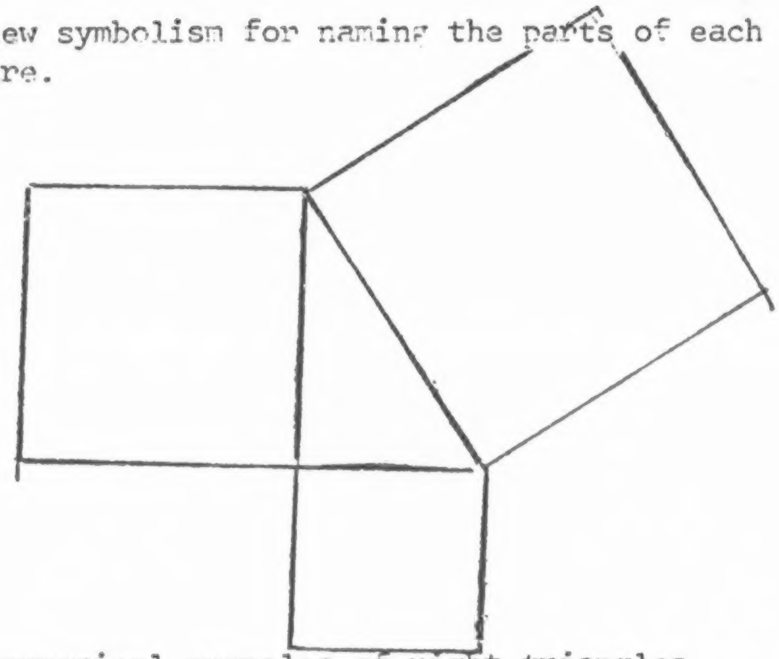
8. 31 cm., 42 cm., 55 cm.

9. 20 mm., 52mm., 48 mm.

10. The three sides of a triangle measure 60 mm., 144 mm., and 156 mm.

Lesson 1 (cont.)

2.
  a. Review symbolism for naming the parts of each square.



  b. Use numerical examples of right triangles with integral side measures.

    i. 3", 4", and 5"
    ii. 6", 8", and 10"
    iii. 5", 12", and 13"

  c. Students should verbally state the meaning behind the Pythagorean theorem.

G. Classwork and assignment: Hectographed sheet.
  1. Find the measures of all sides of each right triangle.
  2. Is each of the following sets of measures that for the sides of a right triangle?

Lesson 2

I. Pythagorean theorem and its applications.

  A. Quiz
    1. Write the formula for the Pythagorean theorem
    2. Using the tables of squares, find the squares of:
      a. 65 b. 433 c. 912

Lesson 2 (cont.)

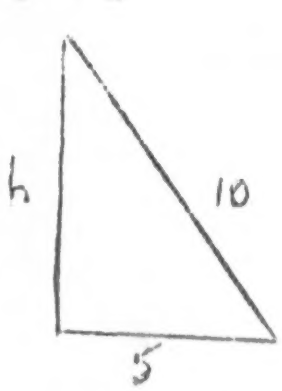
3. Using the tables of square roots find the square root of:
   a. 41 616 b. 539 c. 866.761

B. Review homework
   1. Include a brief review of the area formulas for right triangles.

C. Application of the Pythagorean theorem to finding altitudes of triangles.
   1. Find the altitude of an equilateral triangle given its side measures.
      a. Properties of an altitude
         i. Bisects the base, perpendicular to base
         ii. All three altitudes are equal
         iii. Method of naming altitudes using subscripts
      b. Teacher demonstrates method to find measure of altitude.
         i. Construct perpendicular from vertex to opposite side, forming two congruent triangles.
         ii. Apply the Pythagorean theorem to one of the right triangles formed.
         iii. Show the 1-1 correspondence with the terms of the general equation of the theorem.

$$c^2 = a^2 + b^2 \; ; \; 10^2 = 5^2 + h^2$$

         iv. Solve for h^2
         v. Using tables, approximate h to nearest tenth.
         vi. Using area of formula, find area of triangle.



   2. Use similar method to find area of an isosceles triangle, given side measures.

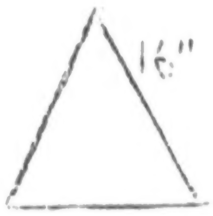
Assignment: Find the altitude and area of each triangle. (Hectographed page).

Name ____________________   Date ____

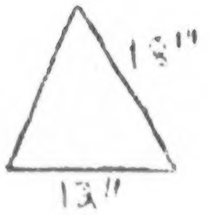
Assignment: Lesson 2

Find the area of each triangle using the given measures.

1. Equilateral triangle



2. Isosceles triangle



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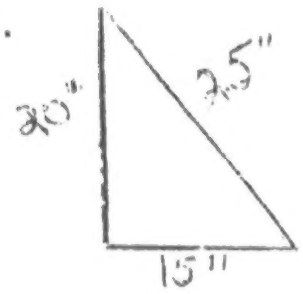

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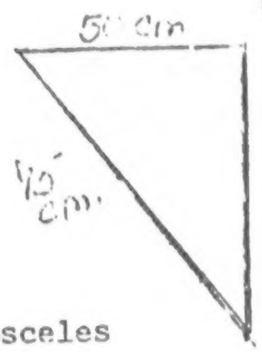

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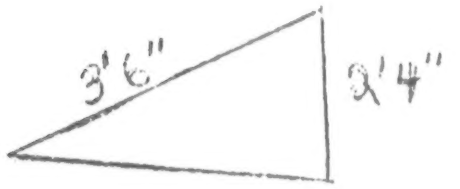
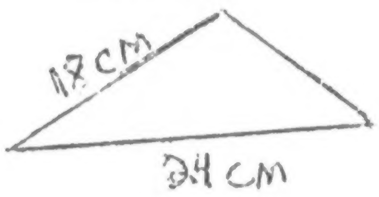


6. Isosceles

triangle



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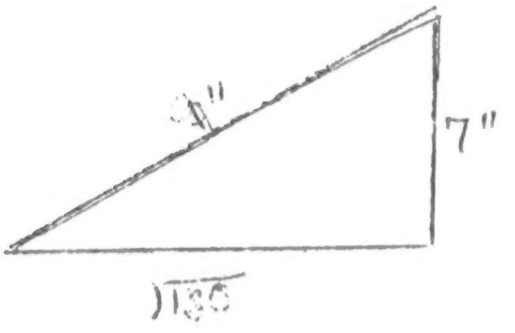

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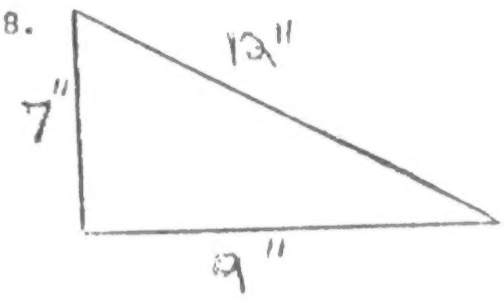


Lesson 3

I. Hero's formula: area of a triangle

- A. Review homework. Discuss:
  - 1. Problems (g) and (h).
  - 2. Need to examine and classify the data given for each problem.
  - 3. Need for planning a method for finding the area.

- B. Review the use of the Pythagorean theorem.
  - 1. To identify right triangles.
    - a. Students should construct the triangle with given parts to confirm that the use of the theorem does correctly identify right triangles.

- C. Hero's formula
  - 1. When given a scalene (oblique) triangle and the length of each side.
  - 2. $A = \sqrt{s(s-a)\ (s=b)(s-c)}$, where:
    - a. A represents the area measure of the triangle.
    - b. s represents 1/2 the perimeter.
    - c. a, b, and c are the sides of the triangle.
  - 3. Review the meaning of the symbols: a) $\sqrt{}$ and b) ________.
    - a. Square root operation (function).
    - b. Vinculum: a symbol of inclusion, indicating order of operation.
  - 4. Demonstrate the use of Hero's formula.
    - a. Find the area of a triangle whose side measures are: a=7 b=5, and c=10
    - b. Must always draw a sketch of the triangle, label and identify all parts.
    - c. Review technique of finding the square root of numbers.

      i. $\sqrt{24}$ ii. $\sqrt{4}\ \sqrt{6}$.

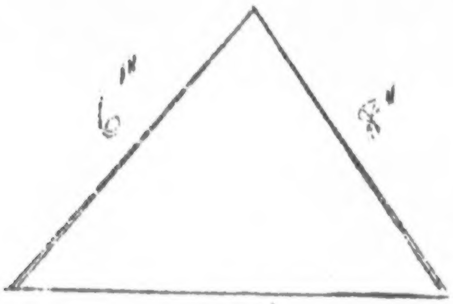
Classwork and assignment: Find the area of each triangle using the given data. (Hectographed sheet).

Name ____________________ Date __________________

Assignment: Lesson 3

Use Hero's formula to find the area of each triangle, unless you can identify an easier method already taught.

1.



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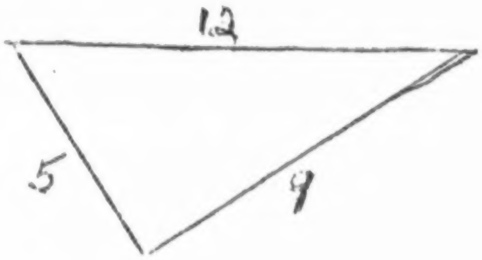

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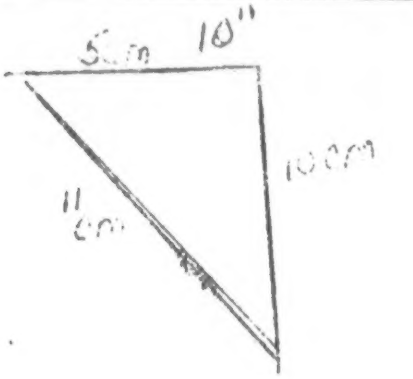
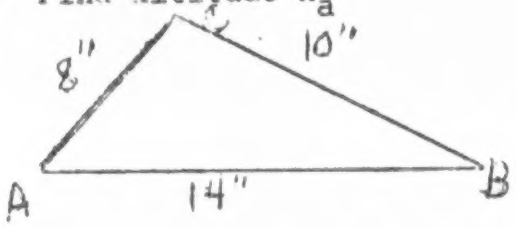
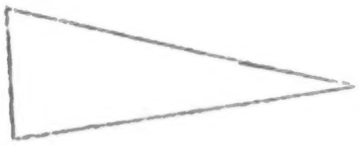


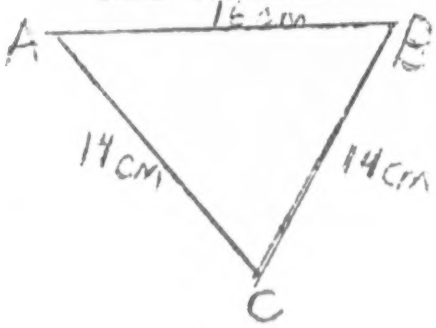
4. Find altitude h_a



5. Find altitude h_b.



6. Find the three altitudes of this triangle.

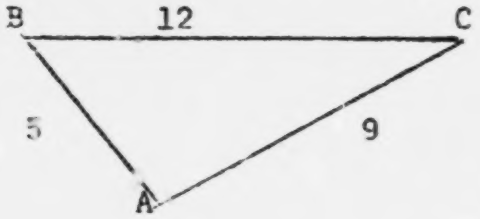


Lesson 4

I. Properties of altitudes of triangles.

A. Quiz
1. Write Hero's formula
2. Describe the information necessary to use Hero's formula
3. Carefully draw the three altitudes of an oblique triangle and name them: h_a, h_b, h_c

B. Review quiz.
1. Review the theorem: the altitudes of a triangle are concurrent.
2. Review the theorem: the area of a triangle can be found by the formula A = 1/2 bh where three choices can be made for the base and corresponding altitude.
   a. Demonstrate the theorems using homework problems as examples.



i. Find the area of each triangle using Hero's formula.
ii. Find the measures of the three altitudes of the triangle by construction.
iii. Find the area of the triangle using the basic area formula: use three different sets of base and altitude.
iv. Compare the results by each method used. They should be the same.

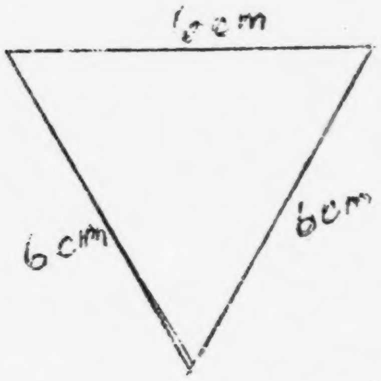
C. Review homework
1. Show that it was not necessary to have used Hero's formula for problems (a) and (e).

Name ______________________ Date ____________

Assignment Lesson 4

1. Determine which triangles are right triangles.
2. Find the area of each triangle.
3. Find the three altitudes of each triangle.

1.



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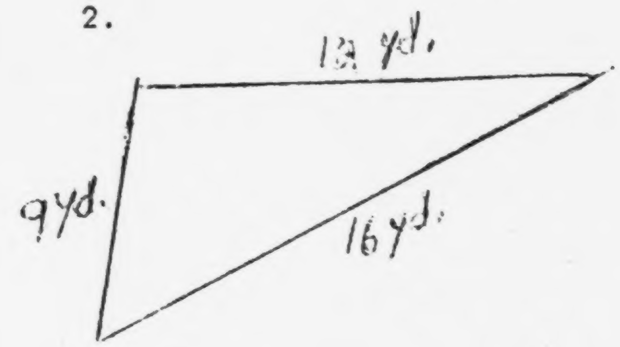

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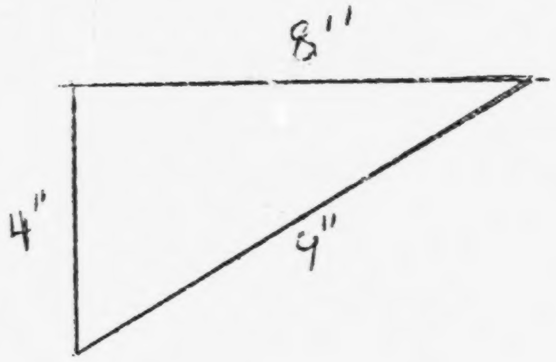

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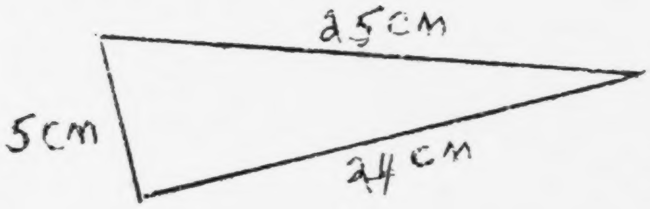


Lesson 4 (cont.)

a. Discuss the techniques to check to see that a triangle is a right triangle.

i. $c^2 = a^2 + b^2$ can be used in a few seconds with the help of a table.

2. It was not necessary to have used Hero's formula for (f)

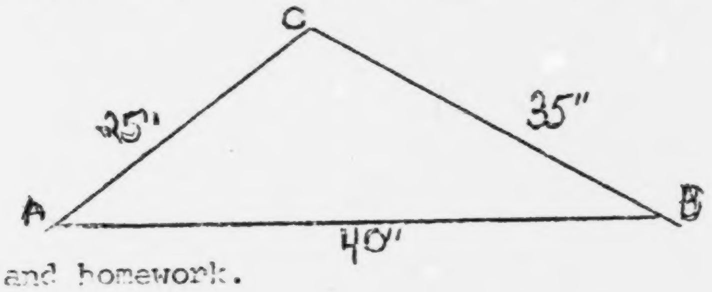
Assignment:

1. Determine which triangles are right triangles
2. Find the area of each triangle.
3. Find the three altitudes of each triangle.
(hectographed page.)

Lesson 5

I. Circles and related lines.

A. Quiz
1. Find the area of the triangle for the measures given.



B. Review quiz and homework.

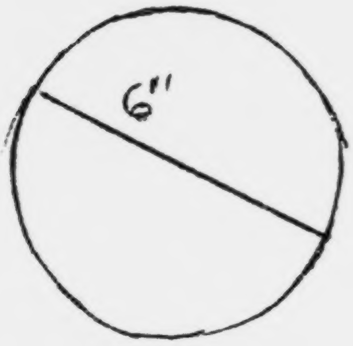
C. Introduce the properties of a circle and related figures.
1. Definition: the set of points on a plane which are at a constant distance from a fixed point in the plane.
2. Related lines.
a. Radius: based upon radial lines rays having
b. Center a common endpoint.
c. Diameter

Name ______________________ Date ____________

Classwork
Assignment: Lesson 5

Approximate the circumference for each of the following circles.

1.



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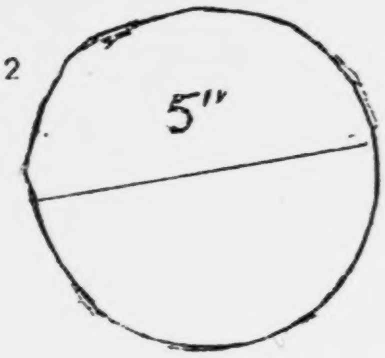

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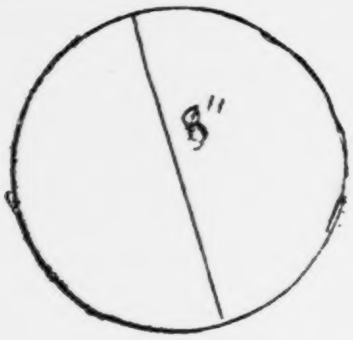

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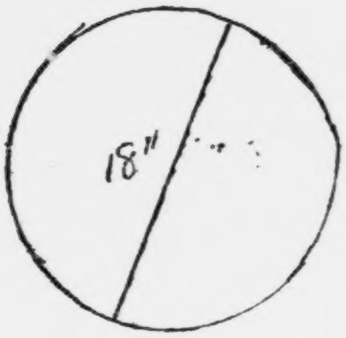

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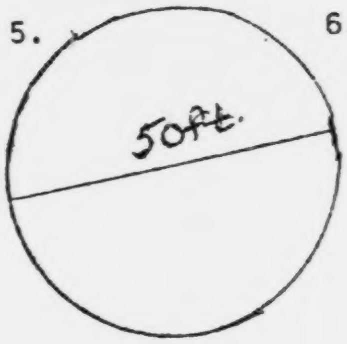

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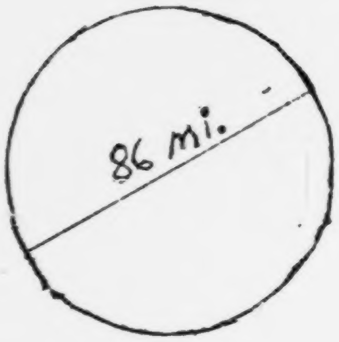
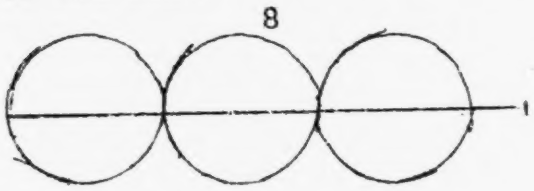


7. Calculate the circumference of each of the above circles, using 3 1/7 or 3.14 as an approximation for .

Lesson 5 (cont.)

- d. Circumference
- e. Chord
- f. Secant
- g. Tangent

3. Parts of a plane related to the circle
   - a. Interior exterior of a circle
   - b. Concentric circles
4. Circumference of a circle
   - a. Relate to the perimeter of a polygon.
5. Relationship of circumference to the diameter of a circle.
   - a. Students construct three different sized disks and use them for their own demonstration.
     - i. Draw a straight line on the board and hold a straight edge against it.
     - ii. Mark the point where the disk first touches the line (or the disk and the line) and roll the disk along the straightedge.
     - iii. Roll the disk until the point on the disk again touches the line. Mark this point on the line.
     - iv. Trace the disk so that the center of the disk lies on the line and the edge of the disk lies on one endpoint of the line. Trace other circles tangent to the first so that the center of each circle lies on the line.



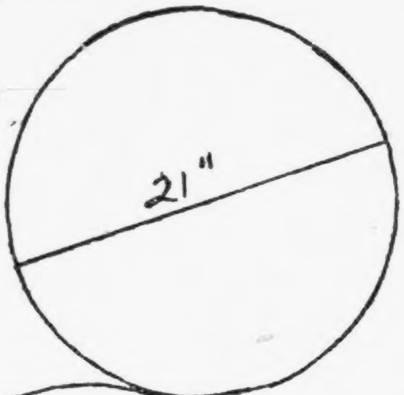
     - v. Establish that approximately 3 1/7 diameters are contained in the circumference of a circle.

Name ________________ Date ______

Assignment: Lesson 5

Find the circumference for each of the following circles.

1.



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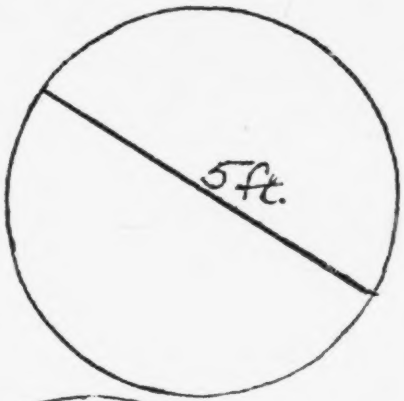

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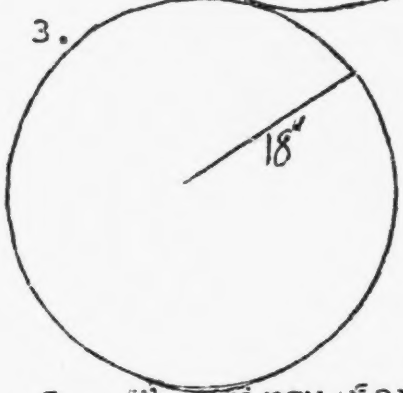

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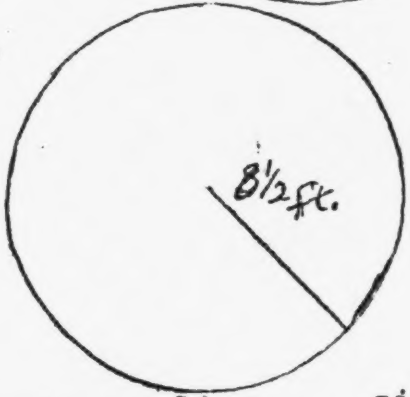


5. The circumference of a circle is [illegible] cm. Using 22/7 for π, find the diameter of the circle.

Lesson 5 (cont.)

- b. Repeat with a circle having a different diameter.
- c. Develop the ratio $\frac{3\ 1/7}{1}$ of $\frac{22}{7}$ as an approximation for .
- d. Establish the decimal equivalent.

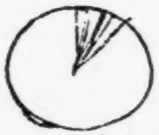
D. Classwork: Approximate the circumference of each circle for the diameter given.

Assignment: Study the new terms
Find the circumference for each of the following circles (hectographed page).

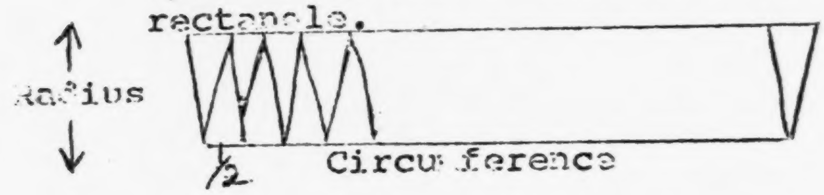
Lesson 6

I. Area formula for circles

- A. Quiz: Name each part of the circle and related lines (hecto)
- B. Review quiz and homework
  - 1. Students demonstrate technique of showing the relationship between circumference and diameter.
    - a. Refer to Lesson 5.
- C. Review definitions of parts related to a circle.
- D. Developing area formula for circles.
  - 1. Cut circle into small sectors.



  - 2. Reassemble sectors into a quasi-rectangular shape.
    - a. Discuss and develop area formula based upon the formula for the area of a rectangle.



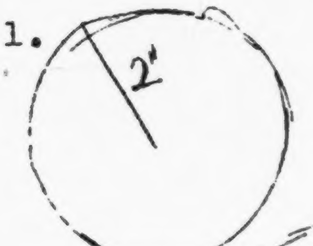
Date ______

Name ________________

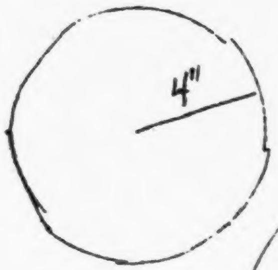
Classwork assignment: Lesson 6

Approximate the area (estimate) of the following circles.

1.



2.



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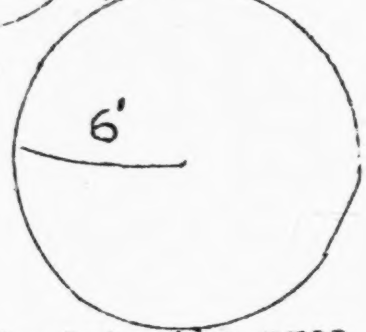

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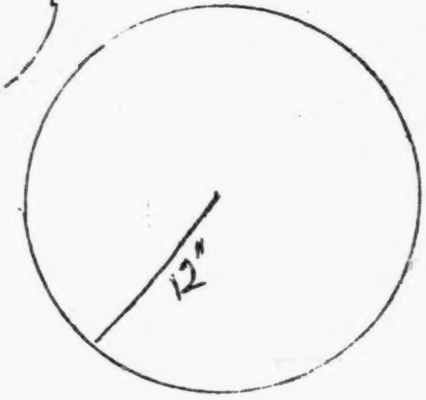
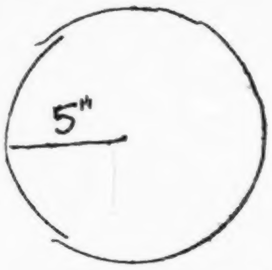


Calculate the area for the following circles using 3 1/7 as an approximation for

5.



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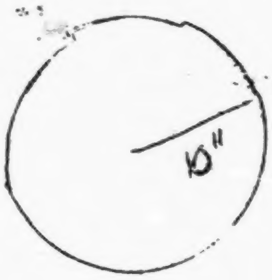
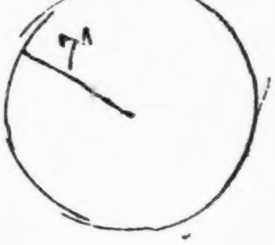
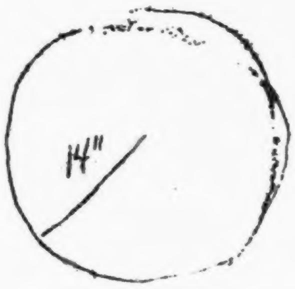


7. What is the relationship between the radius of the circle in #6 to the radius of the circle in #5?

8. What is the relationship of the area of the circle in #6 to the area of the circle in #5?

9. Find the area of each circle. Then compare their radii and areas.



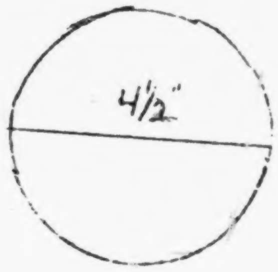


Name ______________ Date ____________

Assignment: Lesson 5

Find the circumference and area for each of the following circles.

1.



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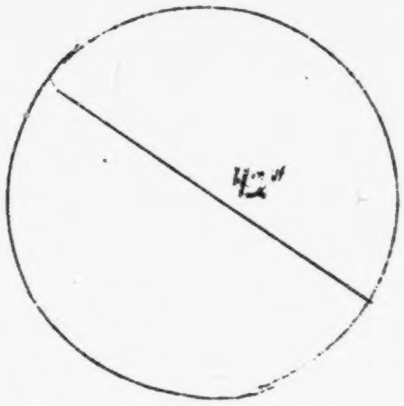

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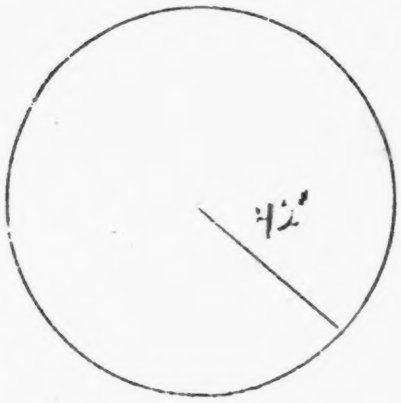

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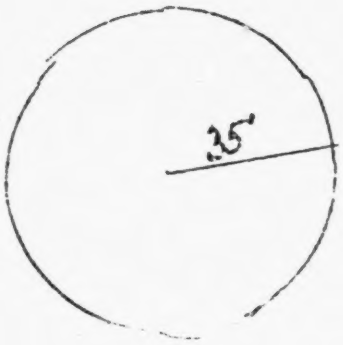

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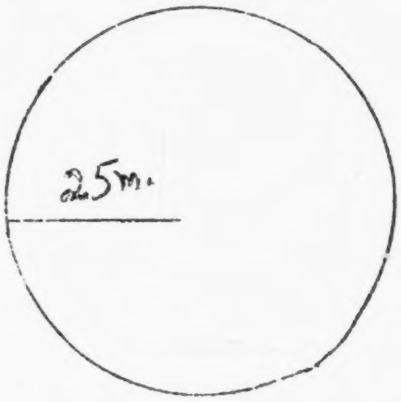


Lesson 6 (cont.)

i. Indicate that sectors can be cut smaller.

3. Teacher explains the development of the area formula:
   a. $A = 1$
   b. $A = \frac{1}{2}$ (circumference) x radius.
   c. Since: $c = \pi d$, then $\frac{1}{2}c = \frac{\pi d}{2}$
   d. $\frac{1}{2}c = \frac{\pi d}{2} r$
   e. Thus: $A = \frac{\pi d}{2} r$
      i. $A = \pi r \times r$.
      ii. $A = \pi r^2$

C. Classwork: Approximate the area of each given circle,

Assignment: Calculate the circumference and area of each circle.

Lesson 7

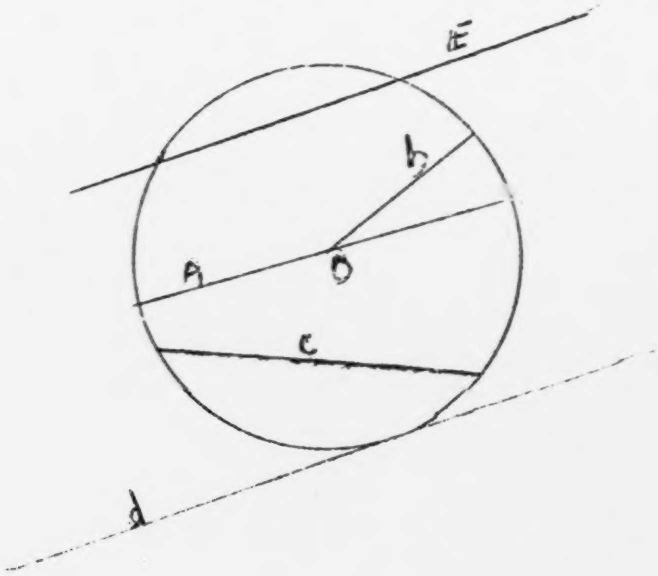
I. Circles and related line segments.

A. Quiz
   1. Draw a circle.
      a. Draw a radius in the circle.
      b. Draw a tangent to the circle.
      c. Draw a chord in the circle.

Name ____________________ Date ________

Quiz: Lesson 6

Write the name of each line or line segment as it is related to the circle.



a ____________________________

b ____________________________

c ____________________________

d ____________________________

e ____________________________

f ____________________________

Name ______________________ Date ______________

Assignment: Lesson 7

Solve the following problems concerned with circumference and area of circles.

1. Find the radius of a circle whose diameter is 28 1/4 feet.

2. Find the circumference of a circle whose radius is 8 1/4 inches.

3. Find the circumference of a circle whose diameter is 12.3 inches. Find circumference to the nearest hundredth of an inch.

4. Find the area of a circle whose radius is $2\frac{1}{2}$ cm.

5. Find the area of a circle whose diameter is 3 1/2 cm.

6. A pipe 6" in diameter supplies water to a small community. Another community is eight times as large in population. What size pipe is required to supply water to the larger community, if their water requirements are eight times as great?

Lesson 7 (cont.)

2. Answer the following:
   a. Is a diameter a chord of a circle?
   b. Is a radius a chord of a circle?
   c. Is a tangent a chord?
   d. Is 3 1/7 the same as ?
   e. Is 3.14 the same as ?
   f. Find the circumference of a circle with a radius of 7½".
   g. Find the area of the same circle.

B. Review quiz
   1. Stress the approximations for .

C. Review homework
   1. When is it most convenient to use 3 1/7

   and when to use 3.14 as approximations for .

D. Review the relationship of areas of circles according to the size of their radii or diameters.
   1. What effect on the area would occur if the radius were doubled?
   2. How many 3/8" diameter (inside) garden hoses are required to give the same cross-sectionl area as one 3/4" diameter hose?
   3. A round pipe has a diameter of 42 inches. How many round pipes with diameters of 7 inches are required to have the same cross-sectional area?

Assignment: Solve the following problems concerned with circumference and area of circles. (Hectograph). Study for a test.

Lesson 8

I. Areas related to circles.

   A. Review lines and line segments related to a circle.

   B. Review formulas for circumference and area of a circle.

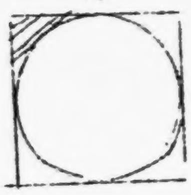
   C. Review homework.

Name ____________________ Date __________

Assignment: Lesson 8

Solve each problem using the area formulas for circles, triangles, and rectangles. Find the area of the shaded portion of each figure.

1.



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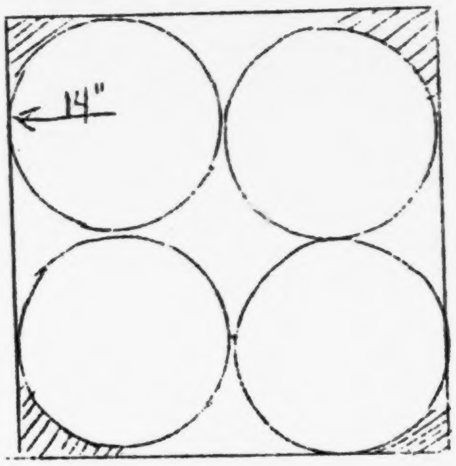

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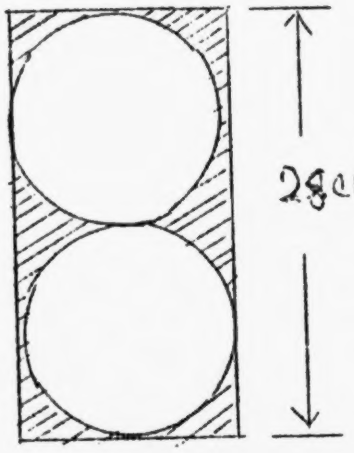

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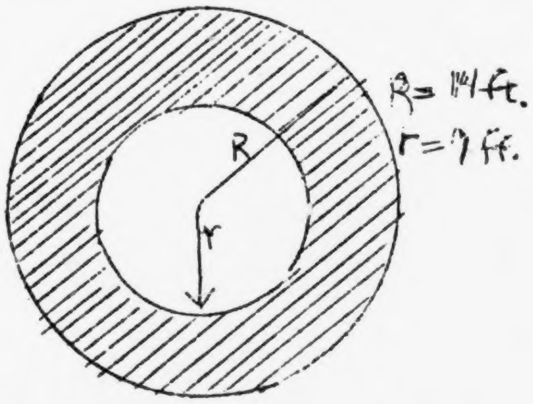

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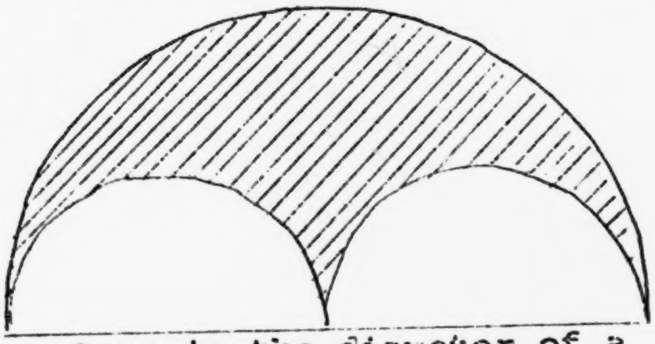
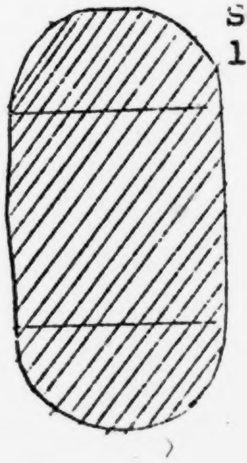


Base is the diameter of a semicircle.
d = 18"

6.

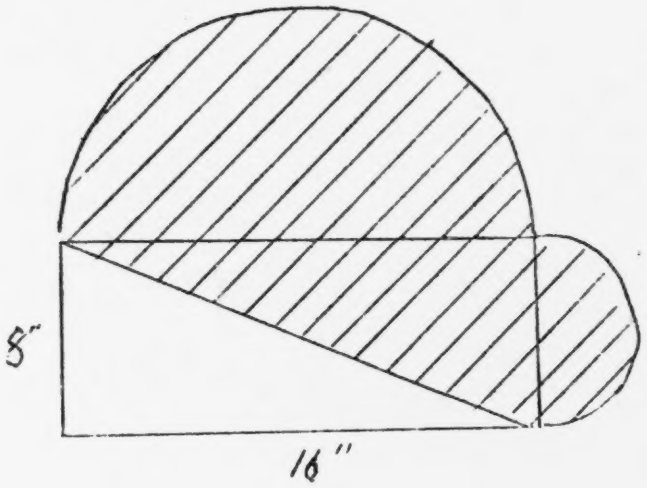


Square with 14" sides

Name ________________ Date ______________

Lesson 8 (continued)

7.



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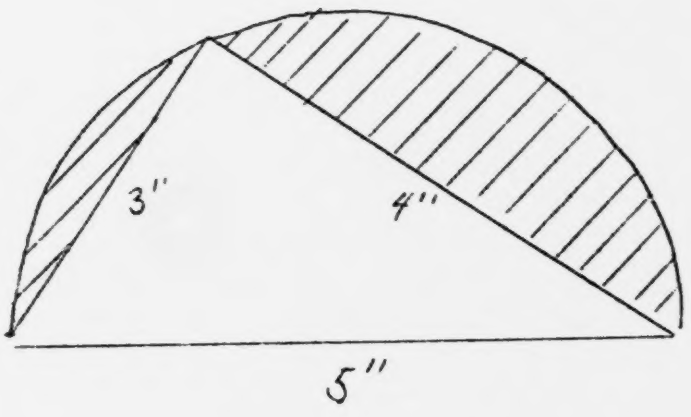
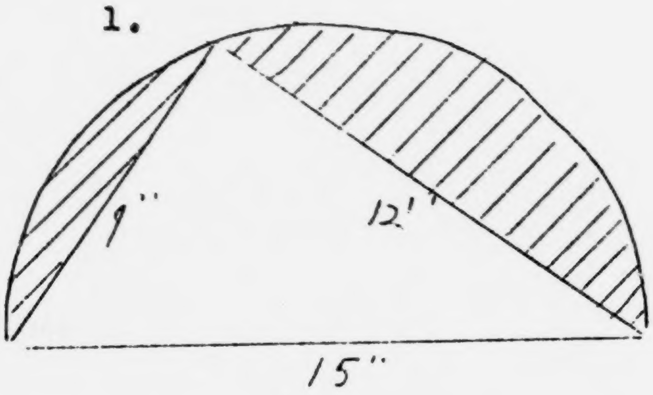


Name ____________________ Date __________

Assignment: Lesson 9

Find the area of the shaded portion of each figure. The outline of each figure is that of a semicircle.

1.



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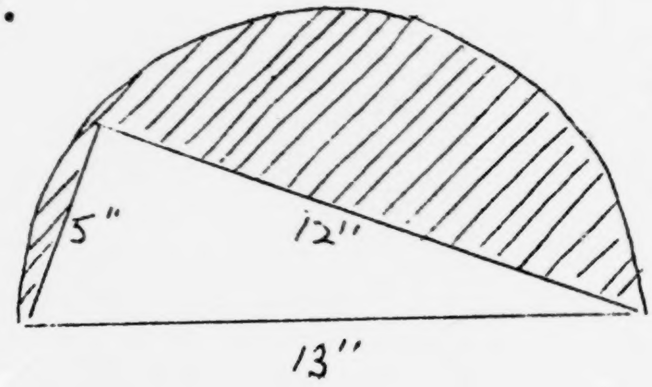


Lesson 8 (continued)

D. Classwork and assignment dealing with irregular areas related to the basic plane figures.
   1. Solve for the area of the shaded portions of each figure. (Hectograph).
   2 Teacher may also draw irregular figures on the board with some dimensions and ask students to estimate areas.

Lesson 9

I. Areas related to the circle.

   A. Review homework
      1. In problem #8 discuss the right triangle.
         a. A triangle inscribed in a semicircle (with its sides passing through the endpoints of the same diameter) is a right triangle.

   B. Classwork assignment:
      1. Solve two problems dealing with triangles inscribed in a semicircle.

Assignment   Study for a test.

Lesson 10

I. Test of material to date (Hectograph)

Lesson 11

I. Review test

II. Review of areas of circles and triangles.

Name ______________________ Date ________________

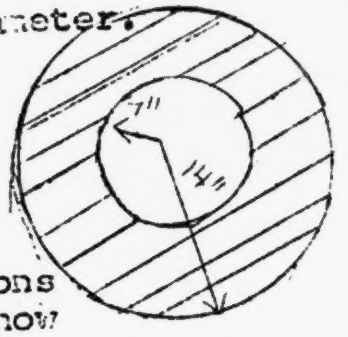
Test: Lesson 10

1. Define circle:

2. Find the circumference of a circle with a 4 inch radius.

3. Find the area of a circle with a 70 foot diameter.

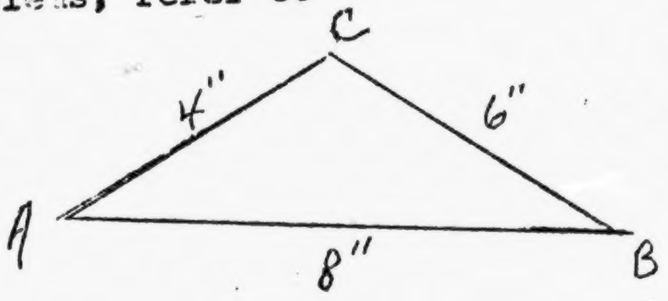
4. Given two concentric circles.
Find the area of the shaded portion.



5. A one-inch garden hose delivers thirty gallons of water per hour. With the same pressure how many gallons per hour will a half-inch garden hose deliver?

6. The circumference of a circle is 47.1 cm.
How long is the diameter of the circle?

For each of the following problems, refer to the figure at the right:
Find:



7. h_a

8. h_b

9. h_c

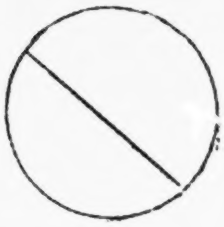
10. The area

Name ____________________ Date ______________

Classwork and homework: Lesson 11

Find the area and circumference of each circle.

1.



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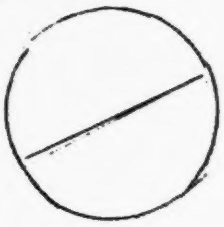
3.



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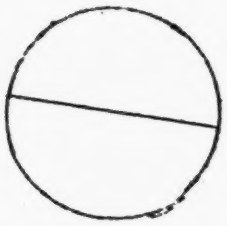

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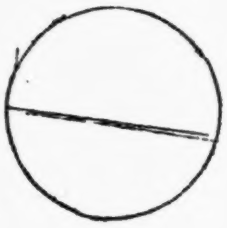
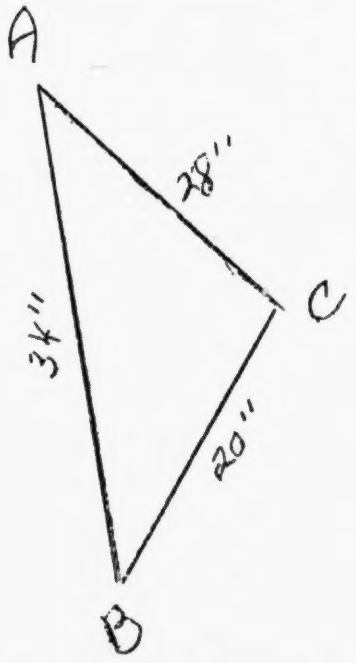


Find the area and altitudes of each triangle.

7.



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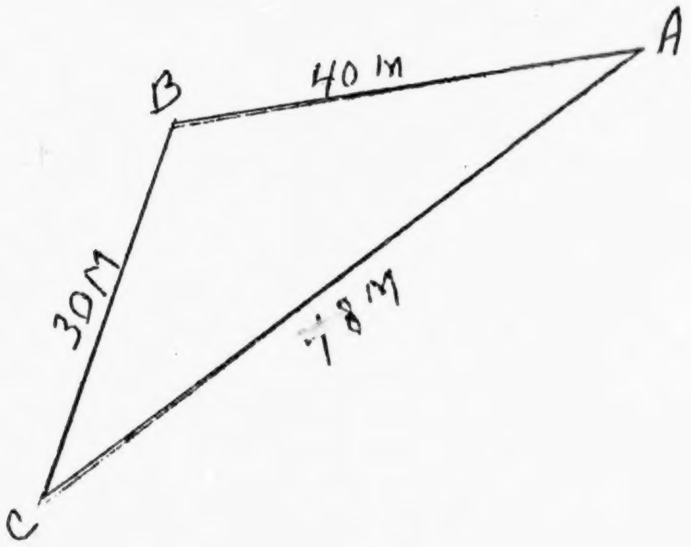


Lesson 11 (cont.)

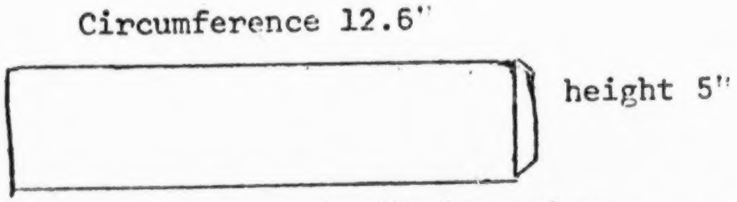
- A. Classwork: (Hectograph)
  - 1. Find the area and circumference of each circle.
  - 2. Find the area and altitude of each triangle.

Assignment: Complete the hectographed classwork assignment.

Lesson 12

I. Surface area of cylinders

- A. Review homework
- B. Introduce right circular cylinder.
  - 1. Using demonstration model, develop the layout for the pattern.
    - a. Disks for bases, perimeter is the circumference of the cylinder.
    - b. Lateral surface is a rectangle, length is the circumference of the cylinder.
  - 2. Layout the pattern for the cylinder which has base of 4" diameter and height of 5".
    - a. Find the circumference of the base.
      - i. By formula
      - ii. Using a flexible strip of cardboard to copy circumference of the circle.
    - b. Layout lateral surface as a rectangle with flap on one end only.

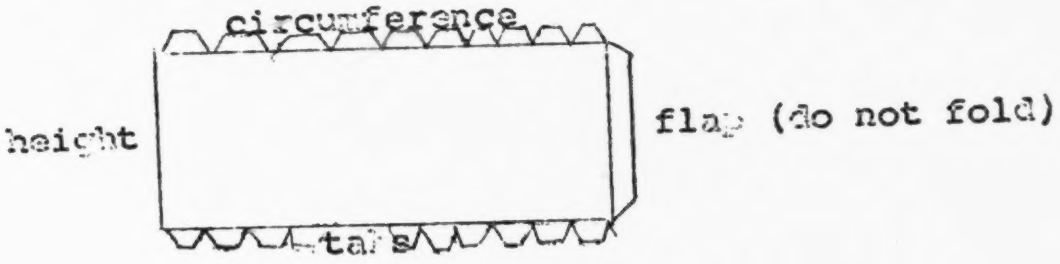


    - c. Cut out two disks of 4" diameter.

Lesson 12 (cont.)

3. Fabricate pattern using scotch tape or white glue.
   a. Do not fold the flap on the stretchout.
   b. Discuss the need for small flaps or tabs on the rectangular layout for attaching the bases.

C. Repeat layout for cylinder with a 3" diameter and 4" height.



1. Students discover need for scalloped flaps on edges.
   a. First layout flaps without scalloping. Try forming.
   b. Students recognize difficulty in forming pattern.
   c. Demonstrate need for cuts on flaps.
2. Cut out bases, form and glue the pattern.
   a. Do not fold the flap on the lateral surface.

D. Classwork and assignment:
1. Layout and fabricate a cylinder from boxboard.
   a. Diameter 2", height 5"
   b. Find the lateral surface area and total surface area.

      i. Find area of parts separately and add.

Lesson 13

I. Patterns for cones

A. Review homework

B. Introduce right circular cone using a demonstration model.
  1. Discuss orthographic projections of a cone.
    a. Top view is a circle, side view is an isosceles triangle.
  2. Identify important parts of cross section.
    a. Radius of base.
    b. Perpendicular height.
    c. Slant height.

C. Use demonstration model to develop the layout of a cone.
  1. Roll cone on its lateral surface to show that the pattern is a sector of a circle.

D. Students, with teacher, construct a cone with given height and radius for base.
  1. Start with two perpendicular intersecting lines.
  2. Mark off radius on one line from point of intersection.
  3. Mark off height on second line from point of intersection.
  4. Join endpoints of these line segments. Measure this line segment as the slant height.
  5. Mark center and, using the slant height as radius, draw a circle.
  6. Circumference of sector for pattern must equal circumference of cone.
    a. Calculate circumference of circle (base).
    b. Use a strip of cardboard to fit around base to copy circumference.
    c. Compare the two results.
  7. Cut our pattern for cone, including flap on side and tabs along base edge.
    a. Do not fold flap on lateral edge.
  8. Base is a disk, with circumference calculated above.

E. Classwork: Follow instructor in constructing a cone whose base diameter is 3" and perpendicular height is 3".

Assignment: Lay out and cut patterns for cones with given dimensions. Use folding boxboard.

Lesson 13 (cont.)

1. Base diameter 8 cm, ; perpendicular height 8 cm.
2. Base diameter 6 cm, ; perpendicular height 10 cm.

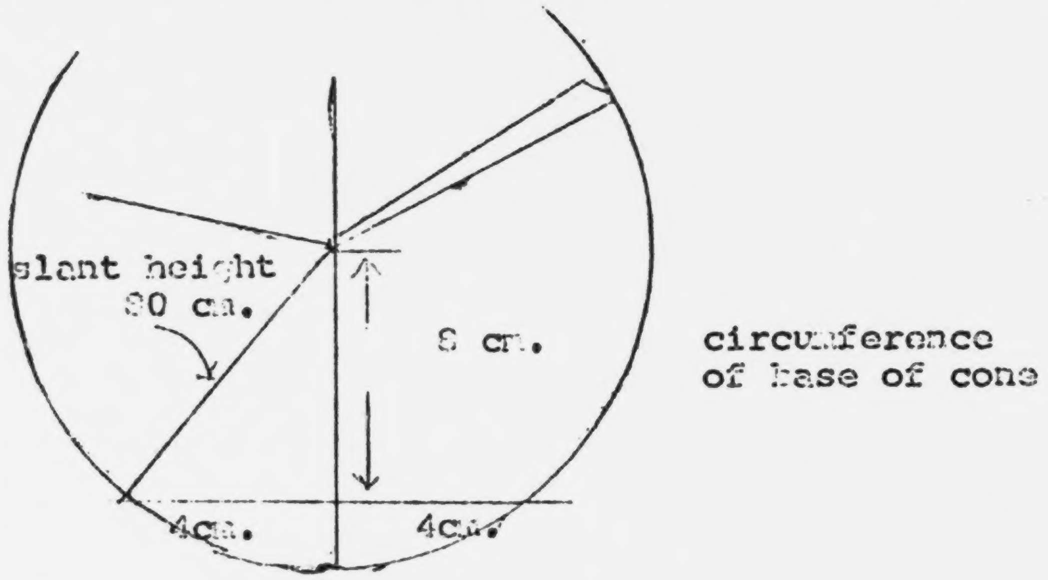
Lesson 14

I. Surface area of cones

A. Collect patterns from homework
   1. Match set of patterns for cones to check sizes.

B. Review layout of pattern for cones.

C. Computing lateral surface area of a cone.
   1. Establish the ratio of circumference of base of cone to circumference of circle having slant height as radius.
      a. Use as examples ratios of and ½.
      b. Multiply fraction by the area of a circle having the slant height as radius.
   2. Students to compute lateral surface area of cones from last night's assignment.
      a. Example: Base radius 4 cm., height 8 cm.



      b. Complete problem on board and have class record work in notebooks.

         i. $\frac{8}{8.9} \quad (\quad 80)^2 = \frac{8}{8.9} . 80$

Lesson 14 (cont.)

D. Classwork and assignment: Draw cross section of a cone. Then lay out pattern for cone and base. Compute the lateral area and base area of each cone for the given dimensions.

1. Diameter of base is 4 slant height is 8
2. Diameter of base is 6 cm. perpendicular height is 3 cm.
3. Diameter of base is 3 cm. slant height is 6 cm.
4. Slant height is 5 perpendicular height is 4 .

Lesson 15

I. Cones - surface area.

A. Quiz: For a right circular cone with a slant height of 13 cm. and perpendicular height of 12 cm. find:
   1. The circumference of its base.
   2. The diameter of its base.
   3. The total surface area.

B. Review quiz and homework
   1. Discuss different types of patterns for right circular cones according to the dimensions of the cones.
      a. Compare #2 and #3 from the homework assignment.

C. Classwork: Introduce right rectangular prism as a piece of duct with ends.
   1. Required: Fabricate a rectangular prism whose base is 2 by 2 and has a length of 2'.
   2. How many faces does the prism have?
   3. How many vertices does it have?
   4. How many edges does it have?
   5. Compute the lateral surface area of the prism.

Assignment:

1. Lay out and cut out a rectangular prism with dimensions 3 cm. by 4 1/2 cm. and 8 cm. long. Find its total surface area.
2. Lay out and cut out a pattern for a cone with base diameter of 7 cm. and height of 9 cm. Find the area of the base and lateral surface.

Lesson 16

I. Patterns for a right triangular prism.

A. Review homework.

1. Form rectangular prisms and check measurements by matching heights.

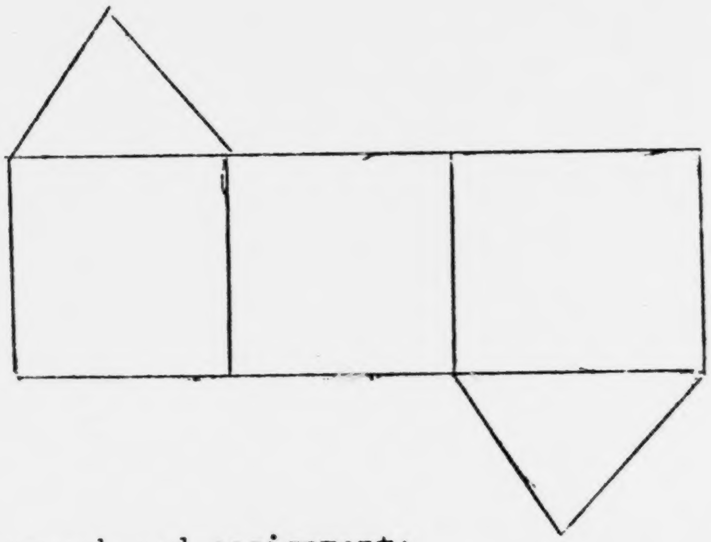
B. Using a demonstration model, show properties of a right triangular prism.

1. Sides are rectangles.
2. Bases in this case are equilateral triangles.

C. Teacher demonstrates stretchout of pattern, noting location of flaps.

D. Classwork: Students construct pattern for a triangular prism whose base edge is 2 (equilateral) and whose perpendicular height is 4 .

1. Students to predict the number of vertices, edges, and faces.
2. Students calculate total surface area.



E. Classwork and assignment·

1. Lay out and cut out pattern for the following geometric solids and find the total surface areas.
   a. Cylinder· base 5 cm. in diameter, height 8 cm.
   b. Rectangular prism· 1 x 1 x 1 .
   c. Triangular prism: base an equilateral triangle with edge of 2 cm., height 2 cm.

Lesson 17

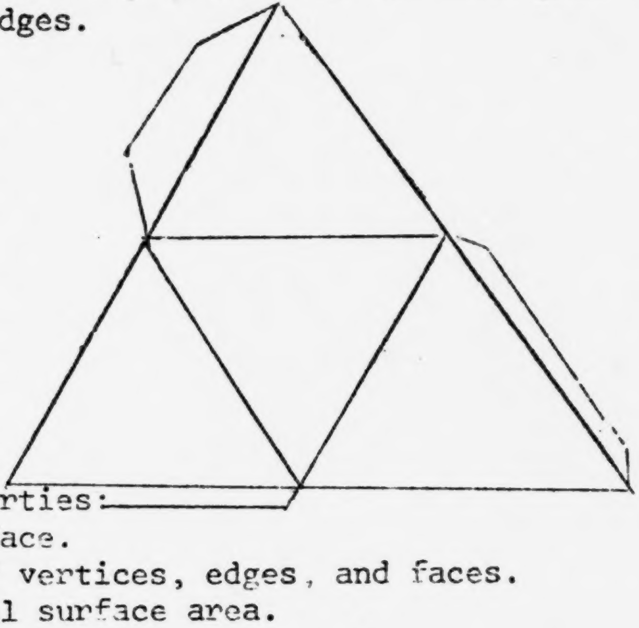
I. Pattern for a tetrahedron.

A. Quiz: Find the number of:
   1. Edges in a rectangular prism.
   2. Vertices in a rectangular prism.
   3. Faces in a rectangular prism.
   4. Faces in a triangular prism.
   5. Vertices in a triangular prism.

B. Review quiz and homework.
   1. Assemble solids from patterns made for homework.

C. Using a demonstration model. show the properties of a tetrahedron.
   1. Definition: A solid having four congruent faces each an equilateral triangle.
   2. Demonstrate: The development of a pattern by:
      a. A stretchout of the pattern for the model.
      b. Construction by compass and straightedge.

D. Classwork: Construct a tetrahedron with an edge of 2
   a. Bisect each side of the triangle.
   b. Connect midpoints, forming four equilateral triangles.
   c. Draw flaps, one flap for each pair of adjoining edges.



2. Discuss properties:
   a. Type of face.
   b. Number of vertices, edges, and faces.
   c. Find total surface area.

      i. By sum of four areas.
      ii. By area of total pattern (less flaps).

Lesson 17 (cont.)

- E. Classwork and assignment: Lay out and cut out patterns for the solids indicated. Find the total surface area of each:
  1. Tetrahedron: Edge 5 cm.
  2. Triangular prism: Base edge 5 cm., perpendicular height 8 cm.

Lesson 18

I. Review properties of a tetrahedron

- A. Students fabricate patterns.
  1. Discuss properites of a tetrahedron.
  2. Check dimenions by matching tetrahedrons.

II. Introduce pattern for a pyramid.

- A. Using a demonstration model, develop the definition of a regular pyramid.
  1. Base is a regular polygon and apex is on a line through the center of the base perpendicular to the base.
  2. The lateral faces are isosceles triangles having a common vertex called the apex of the pyramid.

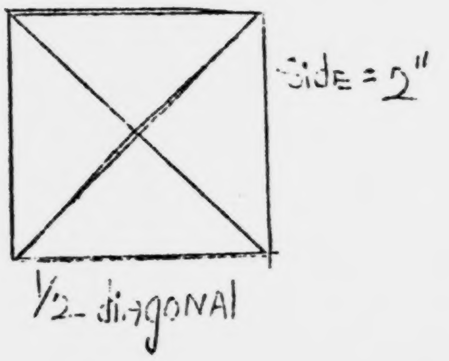
- B. Using a demonstration model, demonstrate the three different heights of a pyramid.
  1. Perpendicular height of a regular pyramid is the distance from the center of the base to the apex.
  2. Edge height is the length of one of the equal sides of the isosceles triangles that constitute a face of the pyramid.
  3. The slant height is the altitude of one of the triangular faces drawn from the apex.

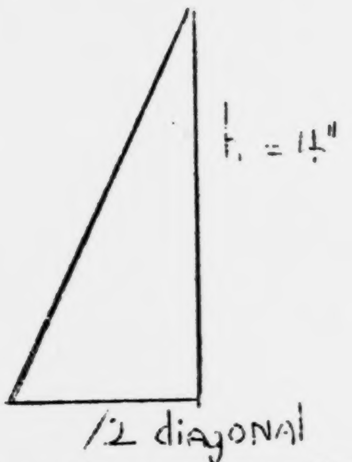
- C. Use the method of triangulation to determine the true length of an oblique line segment.
  1. Find the edge height of a square pyramid whose base is 2" on each edge and whose base is 2" on each edge and whose perpendicular height is 4".

     a. Construct a drawing of the base.
     b. Draw the diagonals of the square.

Lesson 18 (cont.)

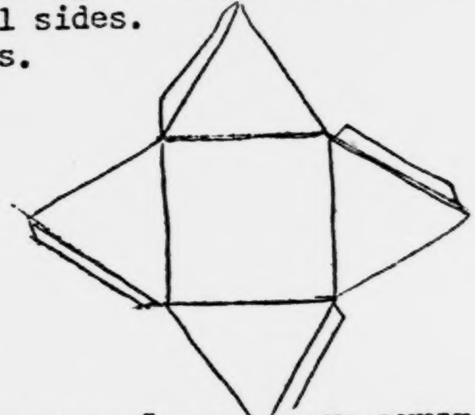
c. Using half a diagonal as base for a right triangle and perpendicular height (4") as the other leg, the hypotenuse will represent the length of the desired edge height.





D. Classwork: Construct a pattern for a square pyramid as described (above).

1. Construct the square base.
2. Construct an isosceles triangle on each side of the square using the edge height as the length of the equal sides.
3. Locate flaps.



Assignment: Construct pattern for a square pyramid with the given dimensions. Edge of base 6 cm., perpendicular height 4 cm.

1. Find the slant height of a triangular face.
2. Find the edge height.
3. Find the total surface area. (Not flaps).
4. Construct the pattern for pyramid and cut it out.

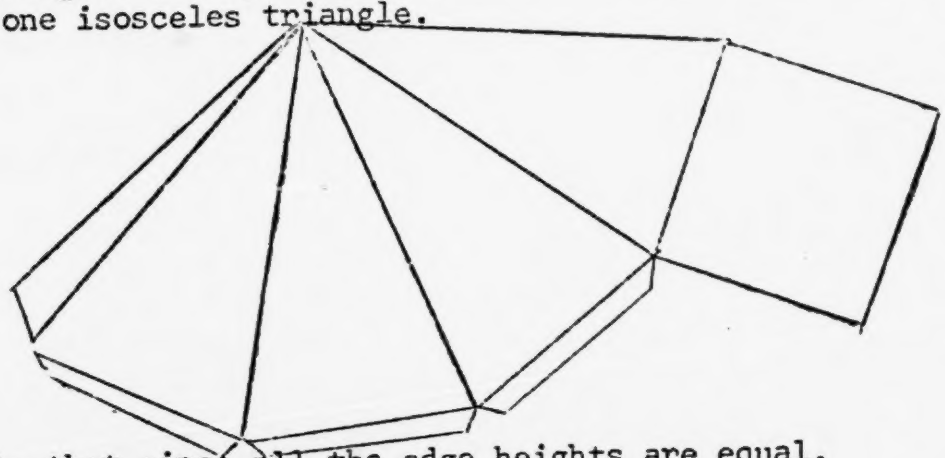
Lesson 19

I. Review parts of a square pyramid.

- A. Use acetate overlay for overhead projector.
  - 1. Pictorial view of pyramid: draw altitudes.

II. A better pattern design for a pyramid.

- A. Using a demonstration model:
  - 1. Cut off the base and cut along one lateral edge.
  - 2. On overhead projector reassemble the four faces meeting at the apex and attach the base to the base edge of one isosceles triangle.



  - 3. Note that since all the edge heights are equal, the vertices at the base edges lie on a circle with center at the apex.
    - a. Note similarity to the pattern for a right cone.

- B. Demonstrate the construction on blackboard.
  - 1. Choose an appropriate point for the apex. Name it A.
  - 2. Set compassopening to the edge height.
  - 3. With point A as center draw a circle with this setting.
  - 4. Set compass to length of an edge of the base of the pyramid.
  - 5. From a convenient point on the above circle mark off four equal arcs of the circle.
  - 6. Draw the chords of these arcs, forming the bases of four isosceles triangles.

Lesson 19 (cont.)

7. Construct a square on the base edge of one of the triangles.
8. Draw the flaps, making one flap for each pair of adjacent edges.

C. Compare the two methods for ease of fabrication.

D. Review homework.

E. Classwork: Construct a pattern for the same pyramid in the homework using the second method of construction.

Assignment:

1. Complete classwork.
2. Construct a square pyramid with a 5 cm. base and 10 cm. perpendicular height.

Lesson 20

I. Introduction to the triangular pyramid.

A. Review identification of parts of a square pyramid.
   1. Number of faces, vertices, edges, apex, three altitudes.

B. Review homework

C. Using a demonstration model, introduce the triangular pyramid.
   1. Differentiate from the tetrahedron, due to perpendicular height variation in the pyramid.

D. Construct the pattern for a right triangular pyramid with an equilateral triangle for a base.
   1. Demonstrate construction using a base edge of 2" and edge height of 4".
      a. Label all vertices.

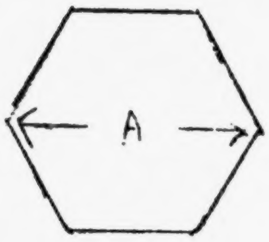
E. Classwork and assignment:
   1. Construct a triangular pyramid with base 4" and edge height 6".
   2. Construct a square pyramid with the same edge dimensions as Problem 1.

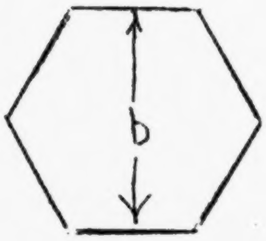
Lesson 21

I. Introduction to the hexagonal prism.

  A. Classwork
    1. Construct a triangular prism with base edge 3" and edge height 5".

  B. Introduce a hexagon.
    1. Definition: A six-sided plane figure in which all edges are equal and internal angles are equal.
    2. Construct an inscribed polygon with edge equal to a radius of the circle.
    3. Explain the dimensions:
      a. Across the points or across the corners.
      b. Across the flats.





  C. Using a demonstration model, introduce the hexagonal prism.
    1. Indicate the two bases as hexagons, six sides as rectangles.
    2. Develop the stretchout on the board by rolling the pattern and tracing the faces.
    3. Note that the hexagonal bases can be made as six equilateral triangles.

Lesson 21 (cont.)

D. Classwork: Construct a hexagonal prism with the distance across the points 6" and height 4".
  1. Construct the lateral faces as adjacent congruent rectangles.
    a. Each base edge is half the distance across the flats.
  2. Construct bases, beginning with one equilateral triangle, to locate the center of the construction circle.

Assignment:

1. Construct a right hexagonal prism: distance across the points is 12 cm. and height is 10 cm.
2. Find the surface area of the hexagonal prism.

Lesson 22

I. Intorduction to the hexagonal pyramid.

  A. Quiz: Tell the number of each part in a hexagonal prism:
    1. Vertices
    2. Faces
    3. Edges

  B. Review construction of a hexagonal prism.
    1. Fabricate the pattern. Review homework.

  C. Introduce the pattern for the right hexagonal pyramid.
    1. Relate to the square pyramid.
      a. Review the three altitudes of a square pyramid (Lesson 18).
      b. Similar properties for the altitudes of the hexagonal pyramid.
      c. Again stress the method of triangulation to find dimensions of oblique line segments.

  D. Classwork: Lay cut on paper a pattern for a hexagonal pyramid having a perpendicular height of 6" and 4" across the corners.

Lesson 22 (cont.)

E. Discuss octagonal prism and pyramid.

F. Classwork and assignment: Lay out and cut out patterns for two congruent hexagonal pyramids and one right hexagonal prism. Dimensions of each: base 8 cm. across the corners and perpendicular height 6 cm.

Lesson 23

I. Introduction to the octagonal prism and pyramid.

A. Check homework: Match a set of patterns to check accuracy.
  1. Students glue a pyramid to each base of the prism.
  2. Check for accuracy of design and neatness of fabrication.

B. Introduce properties of a regular octagon.
  1. Eight sides, vertices, equal angles.
  2. Method of construction.
    a. Draw a circle and locate its center.
    b. Construct perpendicular diameters.

      i. Extend bisectors through center of circle.

    c. Connect consecutive points of intersection of these four diameters with the circle.

C. Classwork: Construct a pattern for a right octagonal prism.
  1. Dimensions 4" across the points, altitude 8".

D. Construct a pattern for a right octagonal pyramid.
  1. Same dimensions as above.
  2. Determine the slant height by triangulation.
  3. Construct the side faces as sectors of a circle.

E. Fabricate each pattern.

Lesson 23 (cont.)

Assignment:

1. Find three different altitudes of the octagonal pyramid in part D above. Check your calculations tomorrow with the measurements of the pyramid.
2. Construct (lay out and cut out) two octagonal pyramids and one octagonal prism with dimensions 4" across the flats and 5" altitude.

Lesson 24

I. Introduction to the right pentagonal prism.

A. Assemble last night's patterns.
   1. Evaluate work for a grade.
   2. Form one compound geometric solid by gluing octagonal pyramids on the bases of the octagonal prism.
      a. Check for tolerance in measurement.

B. Using a demonstration model, introduce the pentagon.
   1. Properties of a regular pentagon.
      a. Five equal sides which form chords of a circle.
      b. Central angles determined are each 72 degrees.
   2. Discuss technique for finding the size of each base angle in one of the five congruent triangles of a pentagon.
      a. $\frac{360 - 72}{2}$.

C. Classwork: Construct a pentagon with edge of 2".
   1. Start with a convenient line segment 2" long.
   2. Using a base of 2" construct an isosceles triangle with base angles of 54 degrees each.
   3. Draw a circle using the apex of the isosceles triangle as center and one of the equal sides as radius.
   4. Complete marking off equal chords and draw chords.

D. Classwork: Construct a pentagonal prism with base edge of 3" and altitude 3".

Assignment:
1. Complete classwork
2. Construct two pentagonal pyramids base edge 3", height 2".

Lesson 25

I. Designing the pattern for a compound figure.

- A. Assemble patterns assigned for homework.
- B. Classwork:
  - 1. Construct a star, using your own individual design.
    - a. Pentagonal prism having squares for faces.
    - b. Two pentagonal pyramids to fit bases of prism.
    - c. Five square pyramids to fit faces.

Assignment: Complete classwork. Record total time required.

Lesson 26

I. Introduction to the octahedron.

- A. Complete homework assignment. Find average time required.
  - 1. Check for accuracy by matching parts.
  - 2. Discuss tolerance when fitting parts.
- B. Using a demonstration model introduce the octahedron.
  - 1. Definition: A solid having eight faces.
  - 2. Develop pattern as two square pyramids with a common base.
- C. Classwork:
  - 1. Construct the pattern for an octahedron with edge 2".
    - a. Construct pattern as that for two square pyramids but do not include the bases.

Assignment: Construct the patterns for three octahedrons. Include gluing flaps. Record the total time required.

1. 4" edge 2. 3" edge 3. 25 mm. edge.

Lesson 27

I. Introduction to the icosahedron.

A. Complete fabrication of octahedrons.
   1. Record average time required.

B. Using a demonstration model, introduce the icosahedron.
   1. Definition: A polyhedron having twenty faces.
   2. Develop stretchout with students.

C. Classwork: Construct a pattern for an icosahedron with 2" edges.

Assignment: Construct patterns for icosahedrons with:

1. 1" edges
2. 35 mm. edges

The mathematics teacher will now place special emphasis on the design and fabrication of Christmas tree decorations for a local business office which had already been contacted by the students from their English class.

Beginning with the next lesson, the class will design and fabricate decorations using aluminum-laminated paper supplied by a local manufacturer. The teacher will emphasize the need for careful work, since the decorations will be put on display in an office building. The students will be expected to record the number of minutes required to complete each geometric solid, so that the class may develop an accurate record of the man-hours of work required for the entire project.

...ing of [illegible] production project.

Students [illegible] ricate patterns of icosahedra.

Introduce [illegible] oncept of a frustum of a solid.
- Illus[illegible] with the concept of a cutting plane
  a. De[illegible] strate with the sawing of a large
  de[illegible] at an oblique angle.
- Demons[illegible] te development of the frustum of a
  right [illegible] are pyramid.
  a. Ha[illegible] two edges of the cut parallel to the
  ba[illegible] of the sides of the pyramid.
  b. De[illegible] op lengths of edges of the frustum
  by [illegible] chniques taught in mechanical
  dr[illegible] ng, beginning with 3-view drawings.

Students [illegible] ign a frustum of a square pyramid.
- Heigh[illegible] pyramid 4", edge of base 2".
- Cutt[illegible] plane not parallel to the base.

Teacher d[illegible] sses plans for study of fabrication of
Christmas [illegible] e decorations.
- Displ[illegible] samples of material used.
- Stress [illegible] eed for care in fabrication.
- Stress [illegible] eed for recording of time required.

- Compl[illegible] construction of pattern for frustum of
  a squ[illegible] pyramid.
- Cons[illegible] set of six (6) each:
  a. [illegible] with 1" edges.
  b. [illegible] with 2" edges.
- Keep [illegible] rd of time used for construction of each
  type [illegible] pattern and total time required.
- Keep [illegible] rd of the number of square feet of
  mater[illegible] required.

Students [illegible] icate patterns of last night's assignment.

- Studen[illegible] record:
  1. Nu[illegible] of each pattern fabricated.
  2. To[illegible] time used to draw and fabricate all patterns.
  3. Nu[illegible] of square feet of material used (also waste).

Lesson 29 (cont.)

B. Teacher emphasizes importance of economy to an industry and this project.

II. Teacher reviews techniques of developing the frustum of a pyramid.

A. Students assigned to design a frustum of a pentagonal pyramid from 3-view drawings.
1. Edge of base 2', height of pyramid 4".
2. Cutting plane parallel to only one edge of the base.

B. Students assigned to draw patterns for a set of six 3" cubes.

Lesson 30

I. Students fabricate patterns of last evening.

A. Students again record time, material, number of patterns.

II. Teacher introduces the parallelepiped.

A. Demonstrates stretchout of pattern.

B. Students assigned to draw patterns for parallelepipeds with 2' edges.

III. Students assigned to draw six (6) patterns for cubes with 4" edges.

Assignment: Complete work assigned in class. Keep record of time, material used, and number of patterns completed.

Lesson 31

I. Students fabricate patterns from last night's assignment.

A. Record time, number of patterns, amount of material used.

Lesson 31 (cont.)

II. Introduce concept of those triangular pyramids which will fit inside a right rectangular prism using all the space inside the prism.

  A. Students assigned to design original patterns for the project.

III. Students assigned to draw a set of three parallelepipeds having 3" edges.

Assignment: Complete work assigned in class. Record time, material, and number of patterns made.

Lesson 32

I. Students fabricate patterns from last evening.

  A. Again record data.

II. Introduce the dodecahedron.

  A. First, as two hexagonal pyramids with a common base.

  B. Construct four dodecahedra with 2" edges.
    1. Discuss construction of the pattern.
    2. Look for shortcuts in construction.

Assignment:

1. Complete patterns for dodecahedra.
2. Construct pattern for a set of three (3) parallelepipeds with 2" edges.

Lesson 33

I. Volume of geometric solids.

  A. Teacher develops intuitive concept of volume.
    1. Using a marble as a unit of volume, estimate the number of these which will fit in a given container (cup, box).

Lesson 33 (cont.)

- a. Use a variety of containers, but use marbles of a constant size.
- b. Test some estimates by filling the container with marbles and counting the number required.

2. Introduce the concept of units of volume as used in industry.
   - a. The number of cans which fit in a carton.
   - b. The number of cars which fit in a car-carrier.
   - c. The number of ice cream containers which fit in a frozen food locker.
3. Introduce the 1" cube as a convenient unit of volume because of the ease in picturing an array of cubes.

B. Display the building of rectangular solids using 1" cubes.
   1. Determine the number of 1" cubes which can be stored in a rectangular prism 3" x 4" x 5".
   - a. Draw a 3" x 4" rectangle on the overhead projector.
   - b. Show how a cubic inch is built upon each square inch of the base.
   - c. Build layers of cubic inches that fill the 5" height.
   2. Students repeat the demonstration with rectangular prism 2" x 3" x 4".

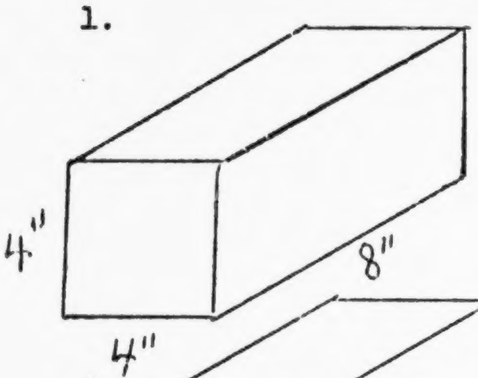
C. Develop the formula for the volume of a right rectangular prism.
   1. Stress the 1-1 correspondence between:
   - a. The number of square inches in the base of the rectangular prism and the number of cubic inches which will fit in the solid.
   - b. The number of inches in the height and the number of layers of cubes which will fit in the solid.
   - c. The volume formula then becomes the product of: (the number of cubic inches of these which will fit in the height of the prism).

Name ______________________ Date ______________

Assignment: Lesson 33

Determine the volume of each of the rectangular prisms. Find the volume in two ways, using two different bases.

1.



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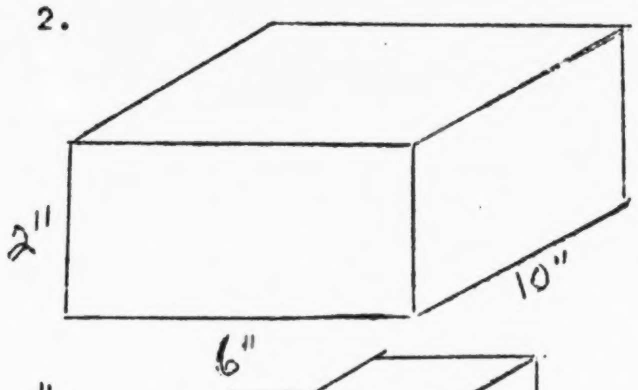

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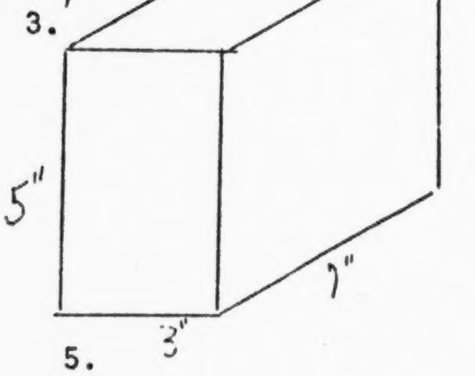

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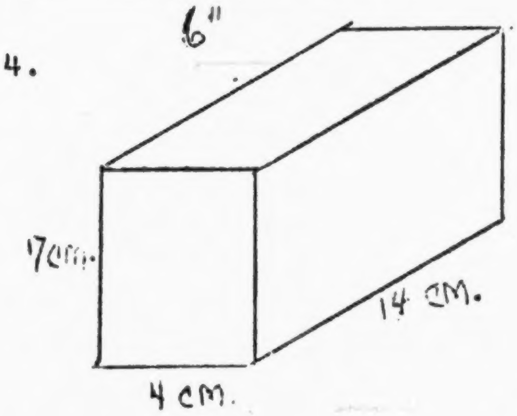

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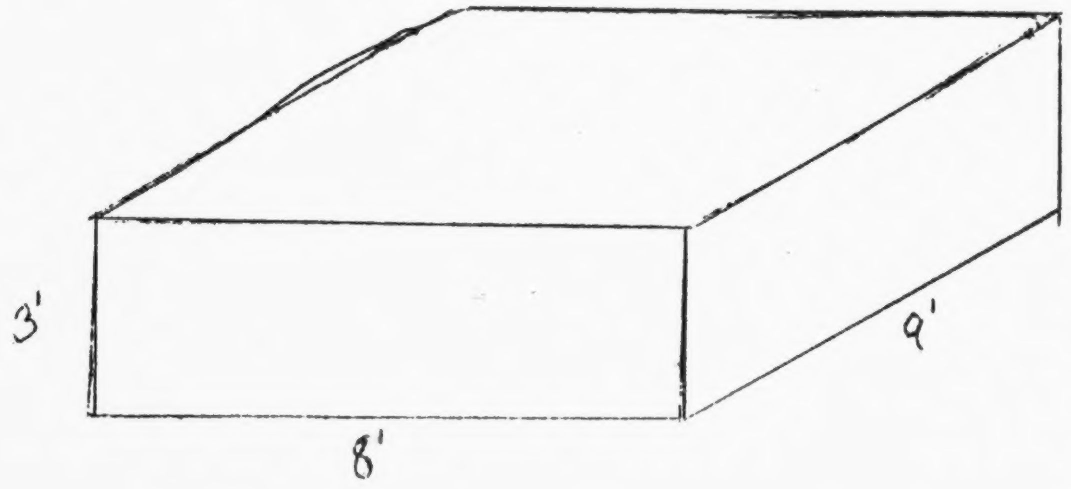


Lesson 33 (cont.)

D. Demonstrate the volume formula.
1. Given a rectangular prism with dimensions: 2" x 4" x 5".
   a. Determine which dimensions shall be used for the base.
   b. Stress the 1-1 correspondence between the area of the base and the number of cubes in one layer.

      i. Thus the conversion: volume equals the product of the area of the base times the height of the prism (in the same basic unit).

2. Choose a different pair of dimensions for a base and develop the volume of the same prism.
3. Students should demonstrate the volume formula for other rectangular prisms.

Assignment: Determine the volume of each of the rectangular prisms for which their dimensions are given. Find the volume two ways, using two different bases. (Hectographed page.)

Lesson 34

I. Geometric solids: volume.

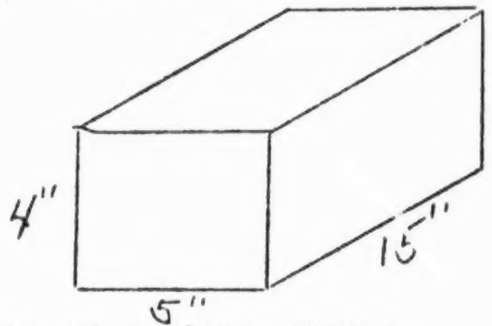
A. Assign project to class - one week's work.
1. Patterns for six basic geometric solids to be drawn and fabricated for display.
   a. Right rectangular prism.
   b. Right circular cylinder.
   c. Right circular cone.
   d. Right hexagonal prism.
   e. Right hexagonal pyramid.
   f. Right octagonal prism.
2. Teacher assigns dimensions with heights the same.
3. All fabricated projects are to be mounted on display board.
4. Material is folding boxboard.

Name ____________________ Date ____________

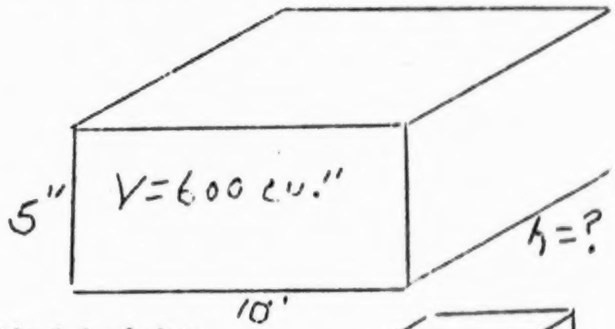
Assignment: Lesson 34

Find the dimensions which may be missing from each object represented.

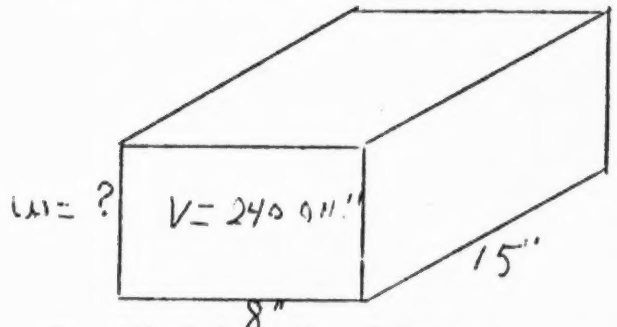
1. Find volume.



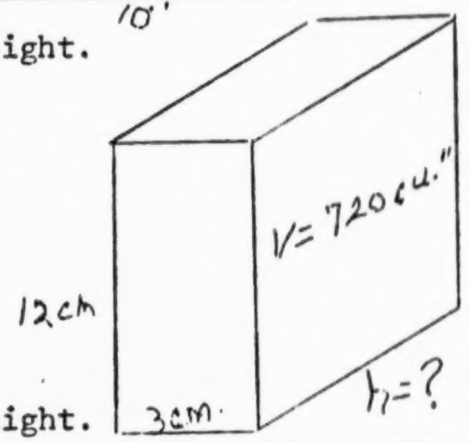
2. Find height.



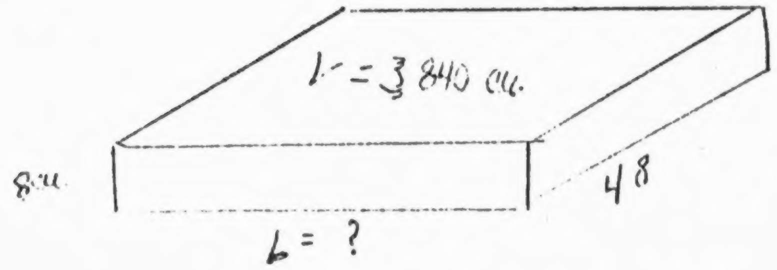
3. Find width of base.



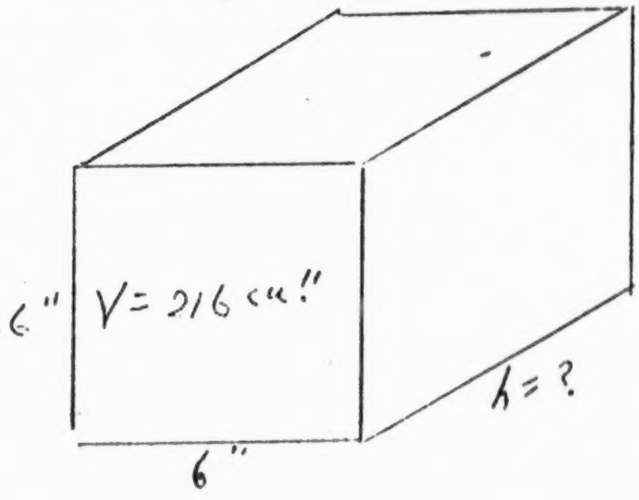
4. Find height.



5. Find length of base.



6. Find height.



Lesson 34 (cont.)

- B. Review homework.
  - 1. Discuss units of volume and their relation to other units.

- C. Formula for volume of a right rectangular prism.
  - 1. V = (area of base) x height.
    - a. All measures in the same linear unit.
  - 2. V = l x w x h.

- D. Classwork on finding missing dimensions of rectangular prism.
  - 1. Develop concept of dividing a number for area by a number representing a linear measure to obtain a number representing a linear measure.
  - 2. Develop concept of dividing a number for volume by a number representing area to obtain a linear measure.

Assignment: Find the dimensions which may be missing from each object represented.

Lesson 35

I. Volume of geometric solids.

- A. Quick review of formula for volume of a right rectangular prism.

- B. Review homework

- C. Volume of other basic prisms.
  - 1. Classwork with demonstration using other right prisms.

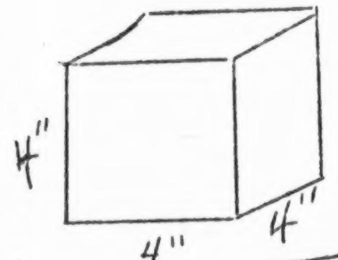
Assignment: Complete hectographed assignment. Continue work on project.

Name ________________   Date ____________

Assignment: Lesson 35

Find the volume of each of the following geometric solids. Use the formula V = (area of base) x height.

1.



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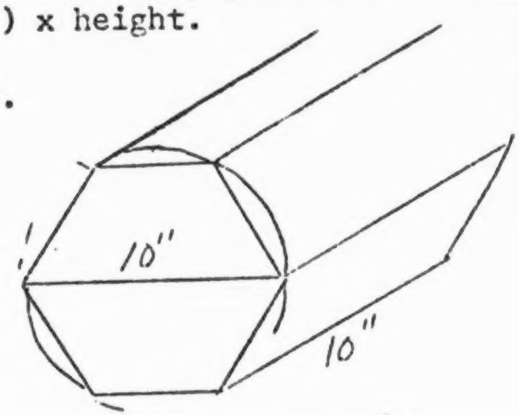

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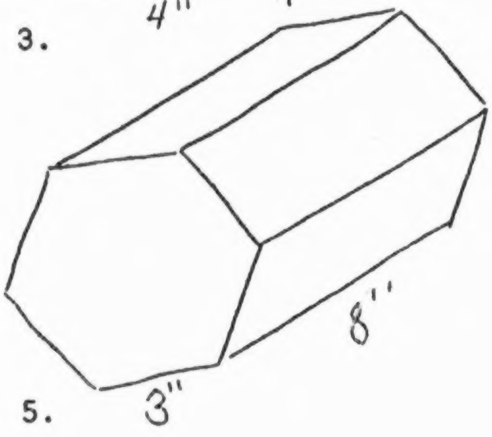

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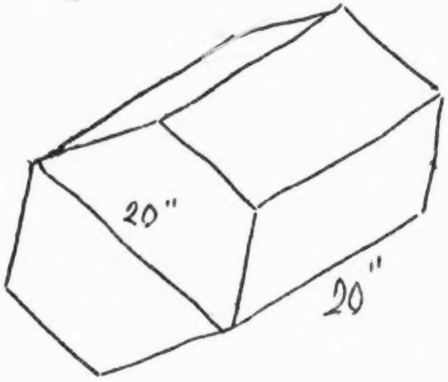

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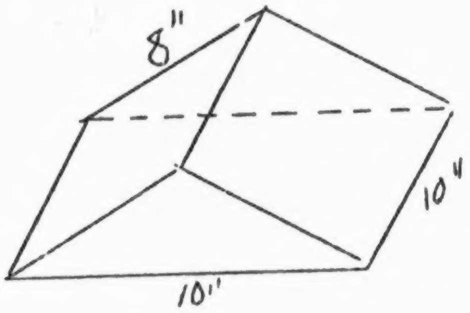

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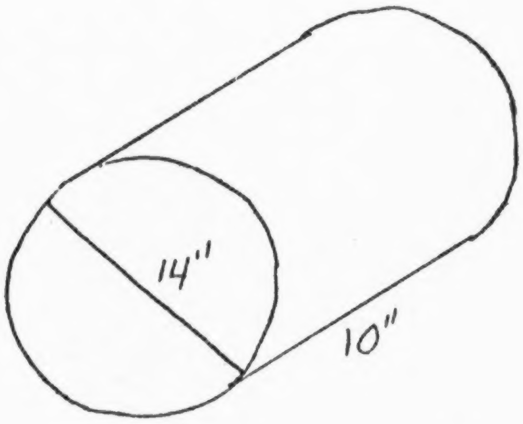


Lesson 36

I. Volume of geometric solids.

- A. Quiz
  1. Find the volume of a rectangular prism 3" x 6" x 9".
  2. Find the measure of a side of a square prism whose altitude is 8 cm. and whose volume is 72 cubic cm.

- B. Review quiz and homework.
  1. Review volume formula for prisms.

- C. Introduce volume formula for a right circular cylinder.

- D. Introduce and develop volume formula for a cone.
  1. Laboratory approach, using previously made models.
     - a. Models of cylinder and cone each with 4" diamter for the bases and 4" altitude.
     - b. Fill cone with sand and pour into cylinder.

       - i. Develop concept that a right circular cone has 1/3rd the volume of a right circular cylinder having the same base and altitude.

     - c. Volume of a cone = 1/3 (area of base) x (perpendicular height).
  2. Classwork: Find the volume of each of the cones for the dimensions given:
     - a. Base diameter 14", perpendicular height 10".
     - b. Base diamter 8", perpendicular height 12".
     - c. Base diameter 12", perpendicular height 8".
     - d. Base diameter 8", slant height 10".

Assignment: Complete classwork and projects.

Lesson 37

I. Volume of geometric solids.

- A. Quiz
  1. Find the volume of a cone 12" across the base and 12" in perpendicular height.
  2. Find the volume of a circular cylinder with a base of 12" and 12" in perpendicular height.
  3. Find the volume of a hexagonal prism whose base edge is 3" and perpendicular height is 8".

Lesson 37

B. Review quiz and homework.

C. Classwork: Find the volume and lateral surface area of each of the following figures:
   1. Right circular cone: base 20 feet and perpendicular height 25 feet.
   2. Hexagonal prism 5 cm. on each edge of the base and 35 cm. in perpendicular height.

Assignment: Complete classwork and continue work on projects.

Lesson 38

I. Volume of geometric solids.

A. Review homework and review for exam.

B. Begin to collect projects and display them on board.

Lessons 39 and 40

I. Collect projects and review for exam.

UNIT 3

DENSITIES OF MATERIALS

Unit 3 of Shop Mathematics I
Densities of Materials

Lesson 1. III

I. Special similar right triangles: Introduction to Trigonometry

  A. Similar triangles.
    1. Illustrate and define similar triangles.
      a. 1-1 correspondence between parts.
      b. Corresponding angles are equal.
      c. Correspondence of sides must be in the same ratio.
      d. Corresponding sides need not be equal but may be.

    2. Property of Isosceles right triangles.
      a. On graph paper draw series of isosceles right triangles.
      b. Students observe properties.
        i. All isosceles right triangles are similar.

    3. Property of 30-60-90- degree right triangles.
      a. Construct a 30 - 60 - 90 degree right triangle on graph paper using a protractor and straight edge.
        i. Label vertices A, B, C.

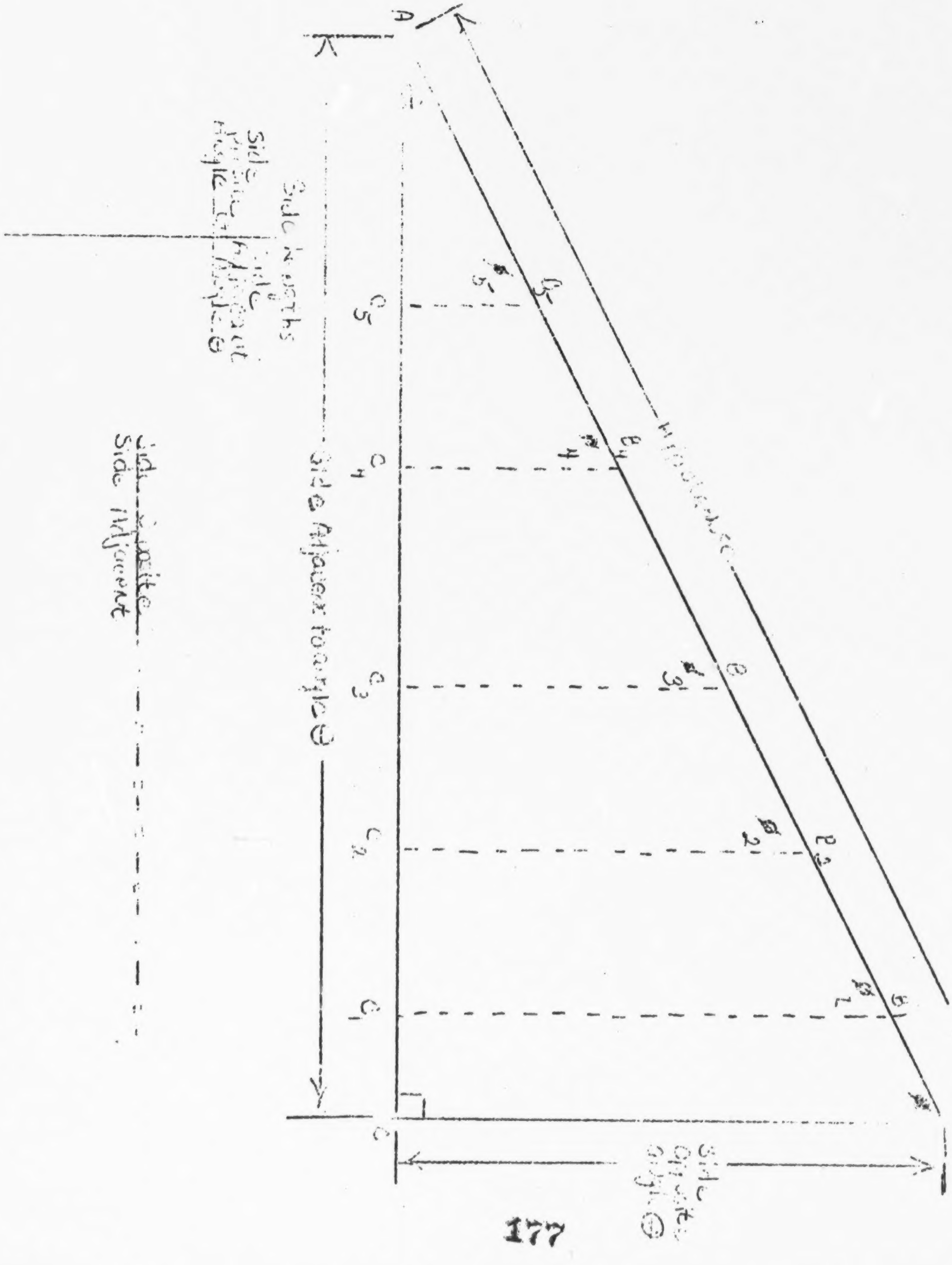
      b. Double the legs of the triangle in 3a, construct triangle label A', B', D'
      c. Triple the length of the legs in triangle 3a, construct triangle label A", B", C".
      d. Cut out smallest (1st constructed) and match angles with other 2 triangles.
        i. Conclusion: Three triangles are similar
        ii. Conclusion: All 30-60-90- degree triangles are similar.

  B. Classwork and assignment

    1. On graph paper draw a right triangle with legs 40 squares long and 20 squares long. Draw the hypotenuse. Draw perpendicular to the 40 unit leg at these distances from the vertex having the acute angle:
       10, 16, 20, 24, 30, 36.

Name________________ Date __________

Classwork, Lesson 1



Lesson 2

I. Ratios of corresponding side-lengths of right triangles.

  A. Review definition of similar triangles.
    1. All isosceles right triangles are similar.
    2. All 30-60-90 degree triangles are similar.

  B. Discuss ratio of the legs of the smallest acute angle in last night's assignment.
    1. Discuss the angle θ (theta) belonging to all the right triangles. $\triangle ABC$, $\triangle AB_1C_1$, $\triangle AB_2C_2$, etc.
    2. $\theta = \theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5$.
      a. Two acute angles of a right triangle are complementary.
      b. Complements of equal angles are equal.
    3. All the right triangles in the figure are constructed for homework are similar.
    4. Compare ratios of the smaller leg to larger leg of each triangle by counting the grid units for each.
      a. Observe: $\frac{BC}{AC} = \frac{B_1C_1}{AC_1} = \frac{B_2C_2}{AC_2} = \frac{B_3C_3}{AC_3} = \frac{B_4C_4}{AC_4} = \frac{B_5C_5}{AC_5}$
        i. In each of these triangles, θ is the smaller acute angle.
        ii. In each triangle, side opposite vertex A is the smaller leg and the side adjacent to vertex A is the longer leg.

      b. Conclusion: in similar right triangles, the ratio of the side opposite the smaller of the acute angles to the side adjacent to the same angle is constant.

    5. Construct a series of similar right triangles by drawing lines parallel to side AC of the triangle in last night's assignment. Then points B_5, B_4, B_3, B_2, B_1 are determined.

      a. Compare ratio of shorter leg to longer leg in each right triangle.
      b. Conclusion: ratios are equal.

  C. Classwork and assignment.

    1. On graph paper construct four right triangles:

   a. Legs: 8 units and 10 units.
   b. Legs: 12 units and 15 units.
   c. Legs: 4 units and 5 units.
   d. Legs: 16 units and 20 units.

2. Measure the angle opposite the shorter side of each triangle.
3. Measure the other acute angle.

Lesson 3

I. Variation in ratio of sides with change in an angle of a triangle

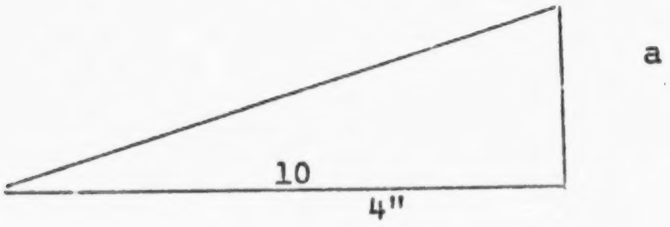
A. Review homework:

1. Note: In similar right triangles the corresponding ratios of the shorter side to larger side (legs) in all the triangles is constant.

B. Develop the concept of the change in ratio of the opposite side to the adjacent side of an acute angle of a right triangle as the angle increases from 10° to 80°.

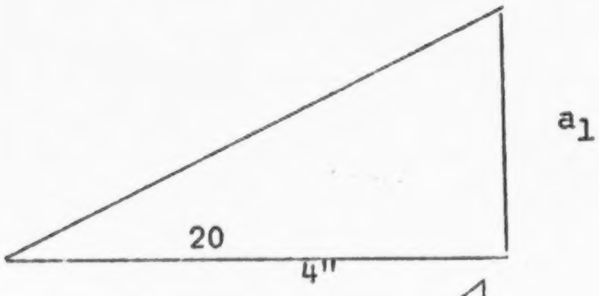
1. Construct a series of right triangles with a constant adjacent side (4") Increase the acute angle 10° in each succeeding triangle.

a.



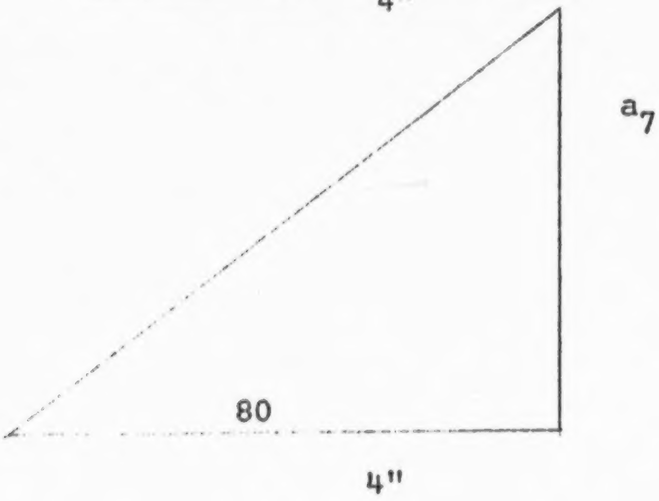
for 10°, ratio $\frac{a}{4''}$

b.



for 20°, ratio $\frac{a_1}{4''}$

c.



for 80°, ratio $\frac{a_7}{4''}$

Lesson 3 (cont'd)

2. Discuss: The change in the ratios of side lengths would be the same regardless of the lengths assigned to the adjacent side.

3. Discuss the other ratios

    a. $\frac{\text{Opposite side}}{\text{hypothenuse}}$

    b. $\frac{\text{adjacent side}}{\text{hypothenuse}}$

4. Make chart with approximate rules for the acute angles 10° 20°, 30°, 40°, ... 80°.

    a. List the ratios of lengths of sides corresponding to the angle sizes.

Name	Ratio	10°	20°	30°	40°	50°	60°	70°	80°
	$\frac{\text{opposite side}}{\text{adjacent side}}$								
	$\frac{\text{opposite side}}{\text{hypotenuse}}$								
	$\frac{\text{adjacent side}}{\text{hypotenuse}}$								

Lesson 3 (continued)

5. Names assigned to the 3 ratios in chart.
   b. sine of an angle
   c. cosine of an angle
   d. tangent of an angle
6. Discuss 1-1 correspondence used in establishing a ratio.
   a. Names of sides of a right triangle and numbers substituted for the names.
7. Introduce trig tables
   a. Examine table for sine of an angle as decimal fractions.
   b. Compose sine of certain angles from table with common fractions previously used.
8. Using table, find ratio for:
   a. sine of 10°
   b. sine of 20°
   c. sine of 30° etc.
   d. compare with common fractions in the table of part 4 above.
9. Discuss approximations of ratios
   a. Due to round off from division.

Assignment: Study the trig ratios

1. Sine of an angle = $\frac{\text{opposite side}}{\text{hypothenuse}}$
2. Cosine of an angle = $\frac{\text{adjacent side}}{\text{hypothenuse}}$
3. Tangent of an angle = $\frac{\text{opposite side}}{\text{adjacent side}}$

Lesson 3 (cont'd)

Using table 11 in text, find the number assigned to

4. sine 15°=

5. sine 24°=

6. sine 39°=

7. sine 45°=

8. sine 66°=

9. sine 4°=

10. sine 88°=

Lesson 4

I. The three basic trigonometric ratios.

A. Quiz
Give ratios according to opposite side, adjacent side, and hypotenuse.

1. sine of an angle =

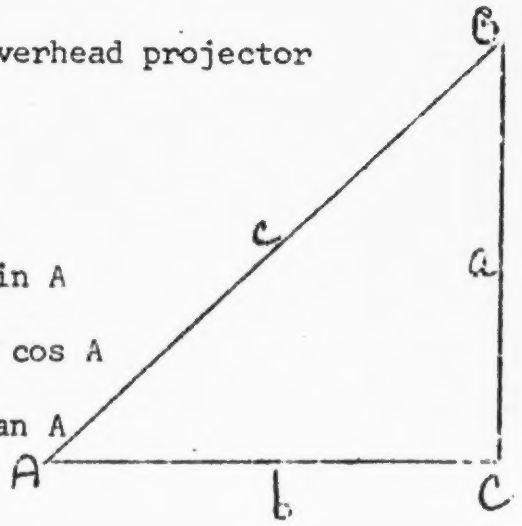
2. cosine of an angle =

3. tangent of an angle =

B. Review quiz

C. Review homework

D. Develope general definitions for 3 trig ratios with abbreviations.



1. Use triangle for model on board or overhead projector

2. Lable vertices and sides

3. Abbreviations

   a. sine of angle A is abbreviated sin A

   b. cosine of angle A is abbreviated cos A

   c. tangent angle A is abbreviated tan A

4. Symbolic representations:

   a. sine A = a/c = $\frac{\text{opposite side}}{\text{hypothenuse}}$

   b. cos A = b/c = $\frac{\text{adjacent side}}{\text{hypothenuse}}$

   c. tan A = a/b = $\frac{\text{opposite side}}{\text{adjacent side}}$

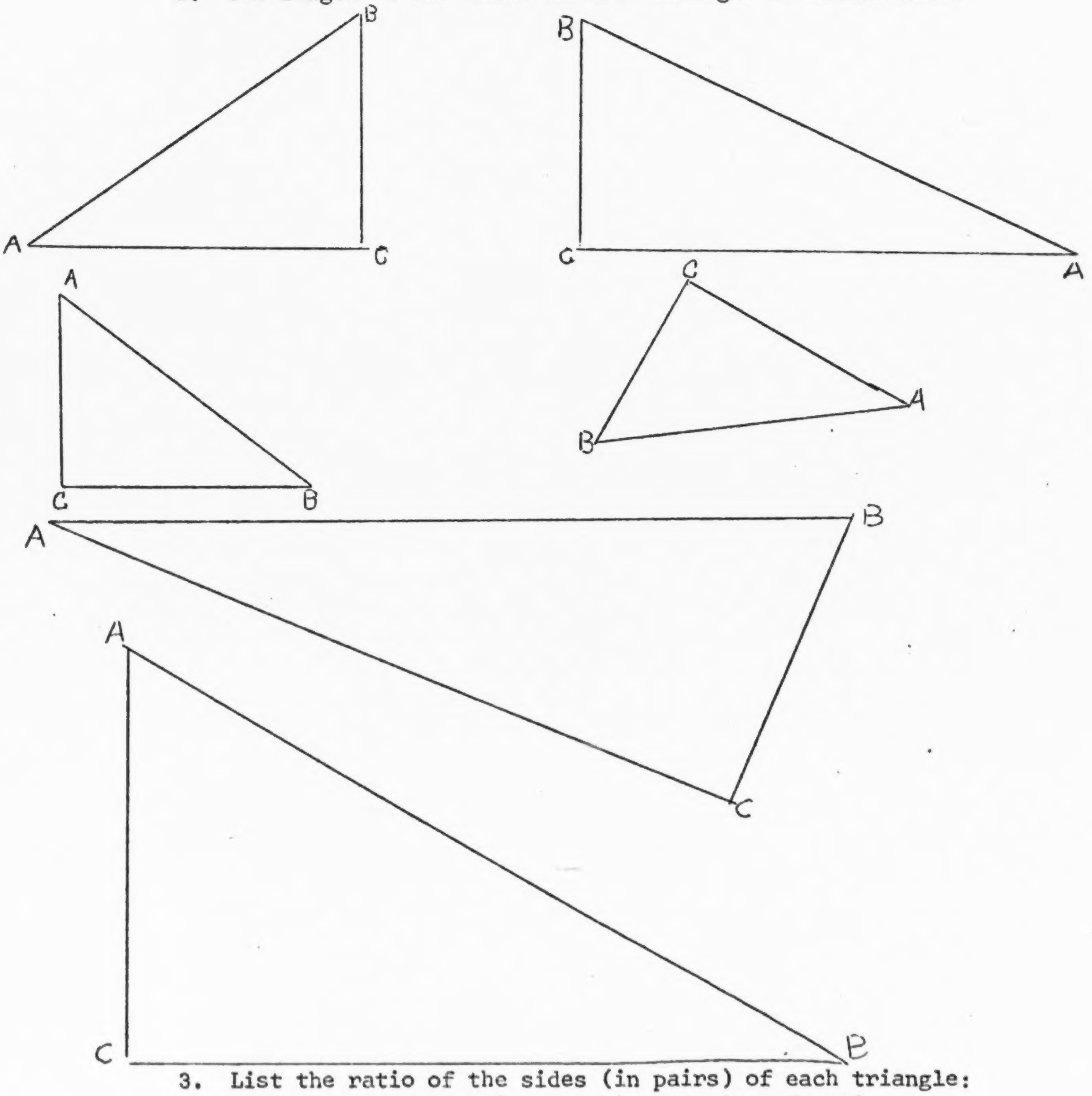
5. Have students in class give answers for

   a. sin B = b. cos B = c. tan B =

Name ____________________ Date _________

Assignment: Lesson 4 List on seperate tables:

1. The size of the acute angles of each given triangle.
2. The length of the sides of each triangle in millimeters.



3. List the ratio of the sides (in pairs) of each triangle:
   a) as common fractions b) as decimal fractions.

Lesson 4 cont'd

E. Classwork: Ditto sheet: List sine, cosine, and tangent for each acute angle in the following right triangle.

Assignment: Study trig ratios
Complete Ditto Sheet

Problem #	Size of angles (Degrees)			Length of Sides (mm.)		
	A	B	C	a	b	c
1.						
2.						
3.						
4.						
5.						
6.						

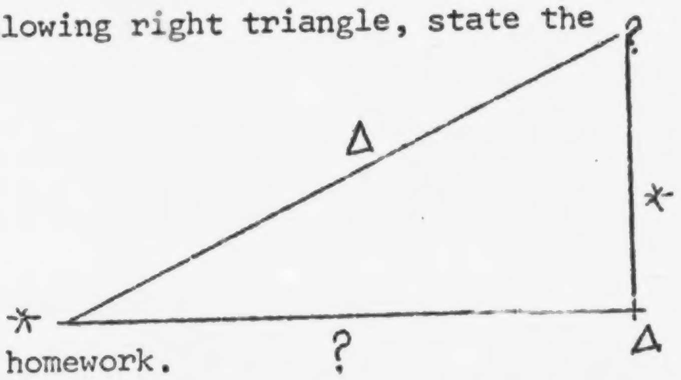
Ratios of sides

As common fractions				As decimal fractions			
Sin A	Sin B	Cos A	Cos B	Sin A	Sin B	Cos A	Cos B

Lesson 5

I. Using the tables of trigonometric functions.

A. Quiz: Given the following right triangle, state the ratios for:
1. Sin angle *
2. Cos angle *
3. Tan angle *
4. Sin ? =
5. Cos ? =
6. Tan ? =



B. Review quiz, review homework.
1. Complete the table of ratios.
a. Review method of changing common fractions to decimal fractions.

C. Introduce: Using the tables of trigonometric functions to find the sine ratio of an angle.
1. Compare decimal quotients for problems to trigonometric ratios in the tables.
a. Show that the decimal numerals express a ratio.

E. Classwork: Practice use of trigonometric tables. Find:
1. sin 18°
2. sin 11° 25'
3. sin 58° 50'
4. sin 73° 33'

F. Finding cosine of an angle, using the trigonometric tables.
1. Relate to finding sine of an angle.
2. Find sin 30°.
3. Find cos 30°.
4. Repeat for 42°, 59°, 60°, 14', 19°, 4'.

G. Finding the tangent and cotangent of an angle, using the trigonometric tables.

H. Classwork and assignment:
1. Complete Slade and Margolis: page 286, #1 to 20.
Reference: Slade and Margolis, Mathematics for Technical and Vocational Schools, 4th edition, John Wiley, 1955.

Lesson 6

I. Using the trig tables.

   A. Quiz: Using table, find:

      1. Sin 18° 21' =
      2. Cos 46° 39' =
      3. Tan 2° 8' =
      4. Cotan 59° 18' =

   B. Review Quiz

   C. Review homework

   D. Classwork: Slade and Margolis
      page 287, 1 to 15

Lesson 7

I. Addition and subtraction of angles

   A. Quiz: Using tables find:

      1. Tan 71° 43' =
      2. Cos 71° 43' =
      3. Cot 38° 1' =
      4. Sin 43° 53' =

   B. Review Homework

   C. Review Quiz

Lesson 7 cont'd

D. Review addition and subtraction of angles whose measure are in degrees and minutes. Classwork:

1.	add	37°41'	41°9'	49°8'	3°2'	22°45'
		18°11'	8°15'	40°52'	8°59'	114°56'
2.	subtract	37°41'	41°9'	49°8'	63°2'	222°45'
		18°11'	8°15'	40°52'	8°59'	114°56'

3. Classwork and assignment: Page 85: 1 to 12 in Slade and Margolis

Lesson 8

I. Using the trigonometric functions

A. Quiz: Add: 1. 6°42' + 12° 9' 2. 46°18' + 39°44'

Subtract: 3. 75°52' − 48°44' 4. 84°19' − 53°37'

Add: 1.	6°42' 12° 9'	2.	46°18' 39°44'
Subtract: 3.	75°52' 48°44'	4.	84°19' 53°37'

B. Review quiz

C. Review homework

1. Sum of interior angles of triangles = 180°

2. Complementary angles

3. Two acute angles of right triangles are complimentary.

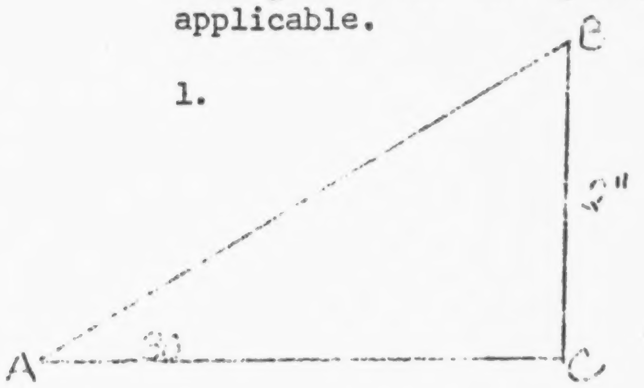
D. Introduce: technique for finding the lengths of sides, size of angles of a right triangle when two parts are given.

Lesson 8 (continued)

1. Construct triangle on graph paper.
    - a. Lay off given side, using grid line.
    - b. Find compliment of opposite angle.
    - d. Construct compliment at one endpoint, right angle at other endpoint.
    - e. Extend sides of angles to form triangle.

E. Classwork: Find the missing parts of the given right triangles. Use the Pythagorean theorem for those where applicable.

1.



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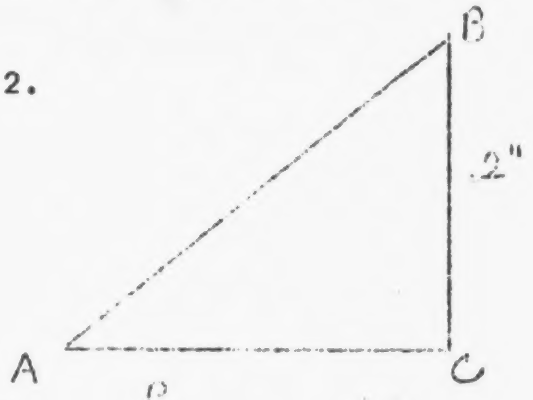

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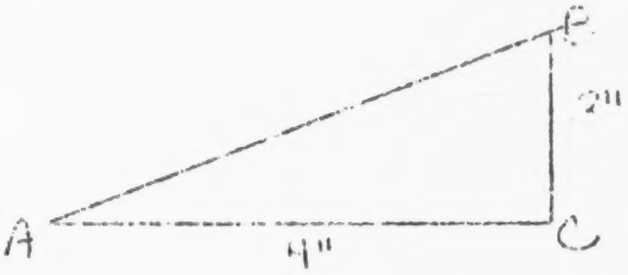

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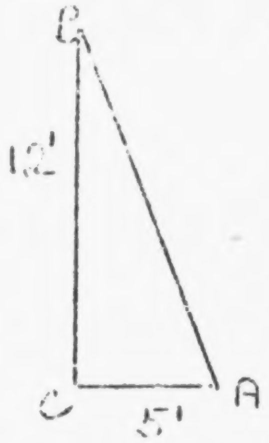


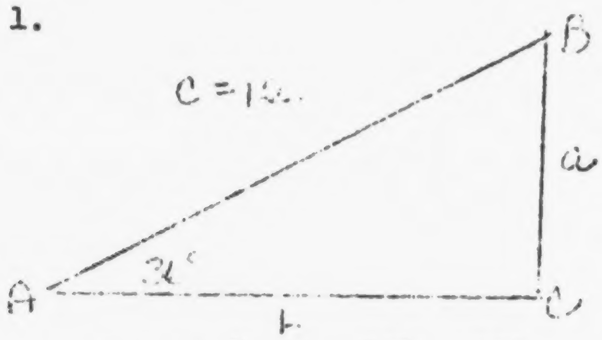
Assignment: Find sine, cosine, tangent, and cotangent ratios for each acute angle in the four triangles above.

Lesson 9

I. Using the trigometric functions: sine and cosine.

A. Review homework. Stress that the process of construction is time-consuming.
B. Finding missing parts of the right triangles:

1.



a. angle B = 90° - 36°

b. check missing sides of triangles.

i "a" is one of the missing sides

ii In what trig function does a appear in a ratio for angle?

iii Discuss one unknown using sin A but 2 unknown using tan and cot of angle A.

iv Use $\sin A = \frac{a}{c}$

a. Calculate for "a"

a. Always start with trig function

$$\sin A = \frac{a}{c}$$

b. Make necessary substitutions

$$\sin 36^\circ = \frac{a}{12}$$

c. Substitute .58779 for sin 36°

d. $.58779 = \frac{a}{12}$ i) Note a is less than 12.

Lesson 9 (continued)

e. $12\overline{)\ a\ }$ with quotient $.58779$ so $a = 12(.58779)$

3. Generalization.

   a. Sine $A = \frac{a}{c}$ thus $.58779 = \frac{a}{12}$

   b. $c(\sin A) = a$ thus $a = 12(.58779)$.

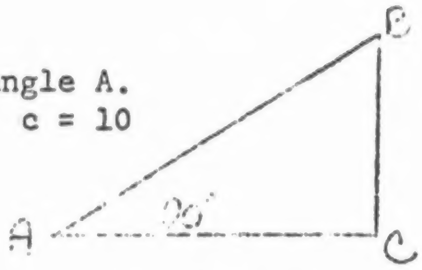
4. Repeat the process to find side b.

5. Find angle B as the compliment of angle A.

C. Repeat with class for this triangle.

   1. Find a, b, and Angle B.

   2. Use relations: $a = c(\sin A)$ and $b = c(\cos A)$.



D. Classwork and assignment (Slade and Margolis)
Page 287 and 288. Do the examples for the figures 237 and 238.
   1. Carefully construct each triangle on graph paper.
   2. Measure all sides and angles.
   3. Then calculate to find missing parts.

Lesson 10

I. Using the sine and cosine functions.

   A. Review homework.
      1. Stress the purpose of the trigonometric ratios: to save time.
      2. Stress sequence of steps in calculating with the trigonometric functions.
         a. Sketch figures and label given parts.
         b. Write the trigonometric functions for which two variables are given and the unknown is in the numerator of the fraction.
         c. Make substitutions.
         d. Solve for remaining variable by multiplication.

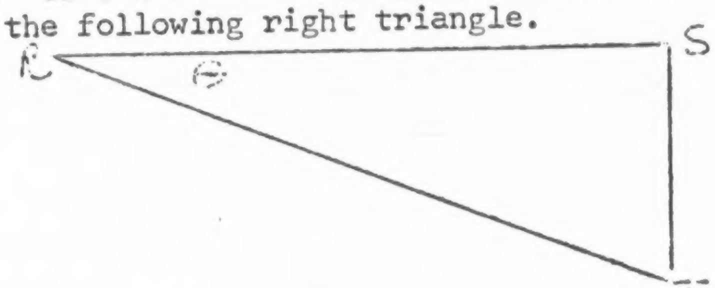
Lesson 10 (continued)

B. Classwork and assignment: Slade and Margolis: Page 290, #7, 8, 9.
   1. Make a sketch of the triangle and label parts.
   2. Use appropriate trig. function to solve.

Lesson 11

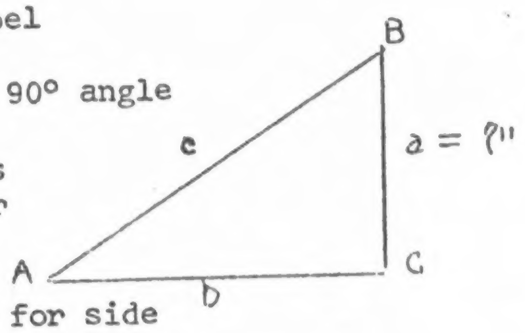
I. Introduction to Secant and Cosecant functions.

A. Quiz: Give the four functions studied to date for angle θ in the following right triangle.



B. Review quiz and homework.
   1. Determine unknown lengths and angles in each problem.
   2. Must draw right triangle and lable parts given.
   3. Determine function for which the unknown is in the numerator and the other parts are known.
   4. Set up each problem.
   5. Find solutions.

C. Classwork: Slade and Margolis, page 290, #1
   1. Given: a = 24° 36' and a = 7" in triangle ABC.
      Find: B, b, and c.
      a. Draw figure and label given parts.
         i. Note: C is 90° angle
      b. Determine functions needed to solve for missing parts.



   2. Using method taught, solve for side c (hypotenuse).
      a. Can solve using Pythagorean theorem after finding side b.
      b. Cannot solve with trig. functions and method taught.

Lesson 11 (continued)

c. Introduce secant and cosecant ratios.
d. Tables for secant and cosecant.

3. Complete solving for side C using secant of angle A.
   a. Secant of angle A is the ratio: $\frac{\text{hypotenuse}}{\text{adjacent side}}$
   b. $\sec A = \frac{\text{hyp.}}{\text{adj.}}$
   c. Thus: $\sec 24° 36' = \frac{c}{7}$
   d. $c = 7(\sec 24° 36')$
   e. $c = 7(1.0998)$
4. Use same plan for cosecant A.
5. Review relationship of six trigonometric functions.

E. Classwork and homework: Slade and Margolis: pg. 290 # 3, 4, 5. Draw figure for each problem.

Lesson 12

I. Using the secant and cosecant functions.

A. Review homework.
   1. Stress sequence of steps in solving for missing parts.

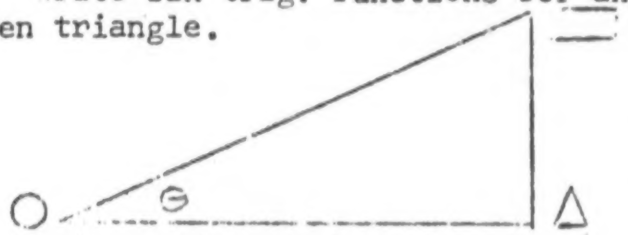
B. Classwork.
   1. Practice for finding missing side lengths using secant and cosecant functions.
   2. Use problems like those in Slade and Margolis, pg. 290 #1-6.

Assignment: Slade and Margolis: pg. 290 #3, 6, 10

Lesson 13

I. Using trigonometric functions to find angle size.

- A. Quiz: Write six trig. functions for angle θ in the given triangle.



- B. Review quiz and homework.
  - 1. Stress need for a drawing of the figure.
  - 2. Label known and unknown parts.
  - 3. Determine function required for solution.
    - a. Use function which will have unknown in the numerator.

- C. Solving for angle when only the sides of a triangle are known.
  - 1. Find angle A when $\sin A = \frac{1}{2}$.
    - a. Change 1/2 to decimal form.
    - b. sin A = .5000
      - i. "Find the angle whose sine is .5000".
      - ii. From the tables under sine we see that 30 corresponds to the sine ratio .5000.
      - iii. Note an 1-1 correspondence.
  - 2. Find missing parts of right triangle. Classwork.
    - a. Given: a = 7.5" and b = 8.75". Find A, B, and c.
    - b. Draw figure, label all given parts, solve.
    - c. Note different approaches to problem: Pythagorean theorem.

Assignment: Slade and Margolis: page 291 (top) #2-10.

Lesson 14

I. Using trigonometric functions to finding angle sizes.

- A. Quiz: Slade and Margolis, page 290 #10.

- B. Review quiz and homework.

- D. Classwork and homework. Slade and Margolis, page 291 (top) # 5. Page 291 (Bottom #1, 2, 3, 4.

Lesson 15

I. Finding angle sizes, given side measures of a right triangle.

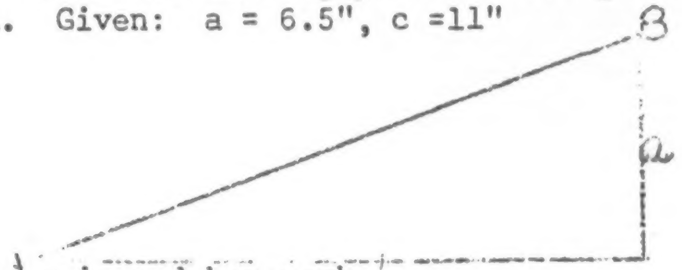
A. Quiz: Slade amd Margolis, page 291 (bottom) #5.
B. Review quiz and homework.
C. Teacher introduces concept of determining angle sizes, given two legs of a right triangle.
  1. Students construct right triangle on graph paper.
    a. Measure acute angles.
  2. Students next establish tangent and cotangent ratios for acute angles, using given measures for legs.
    a. Use tables to find corresponding angle.
  3. Students compare results of (1a) and (2a).

D. Classwork and homework: Slade and Margolis, age 291, bottom #6 to #10 inclusive. Follow plan used in class.

Lesson 16

I. Finding the altitude of an isosceles triangle.

A. Quiz: Find the missing parts of the given right triangle.
  1. Given: a = 6.5", c =11"



B. Review quiz and homework.

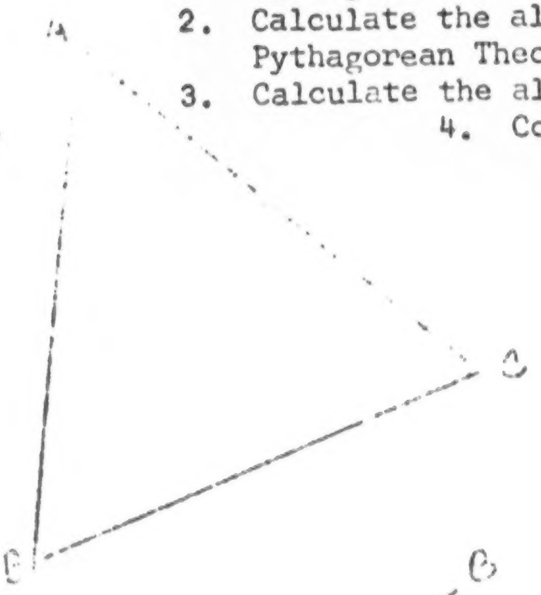
C. Finding the length of the altitude of an isosceles triangle.
  1. Students construct the altitude of an isosceles triangle using compass and straight edge.
  2. Review properties of an isoscelse triangle and the the altitude from the vertex of the angle opposite the base.
    A. Altitude to base bisects base and is perpendicular to the base.

Name ______________   Date ____________

Assignment: Lesson 16

1. Construct the altitude to the base of each isosceles triangle.
2. Calculate the altitude by measuring the sides and using the Pythagorean Theorem.
3. Calculate the altitude by using the trigonometric functions.
4. Compare your results.

1.



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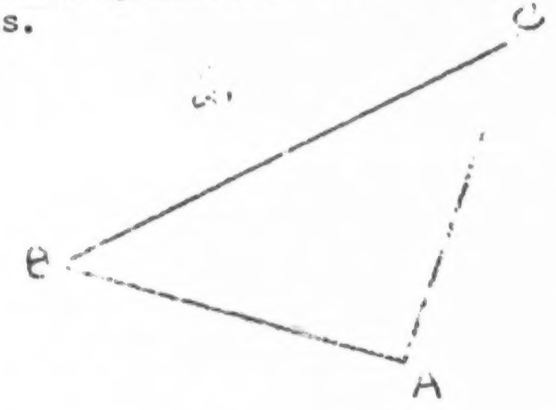

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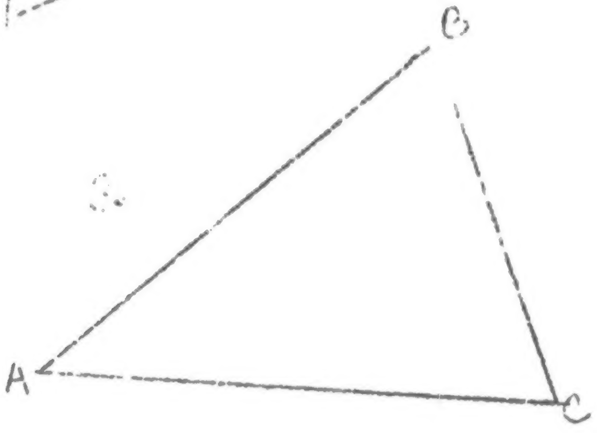

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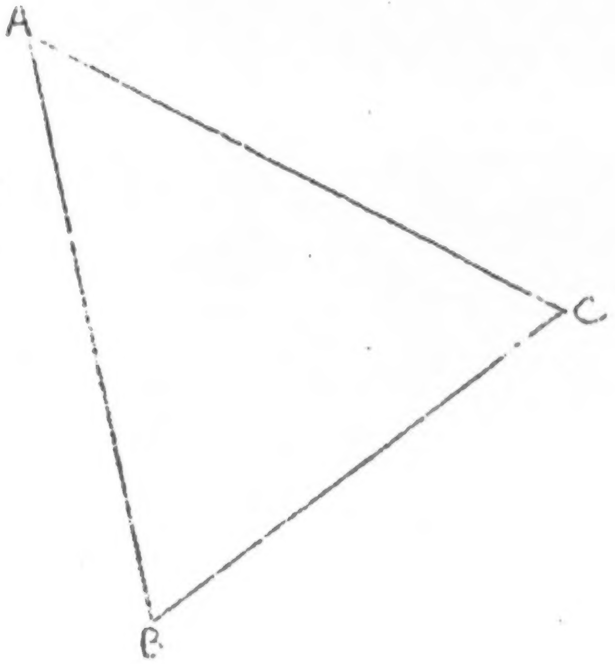

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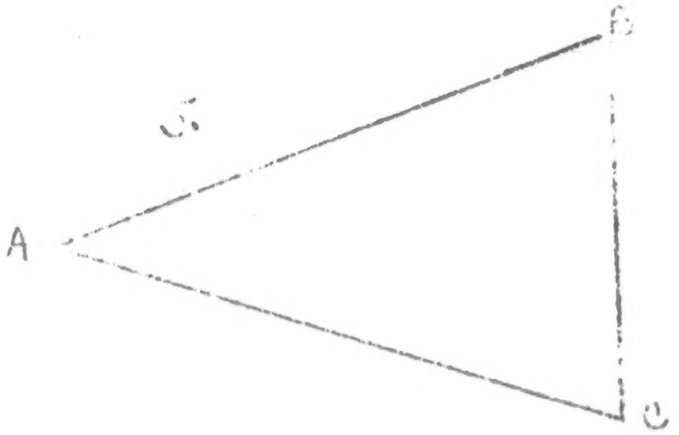

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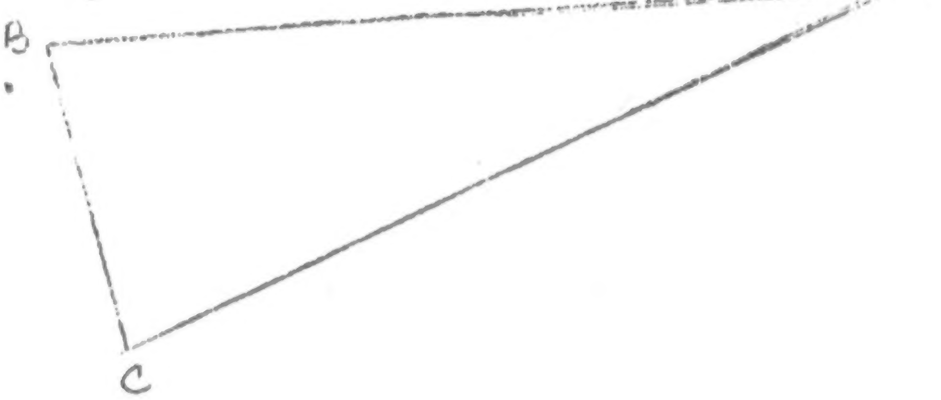
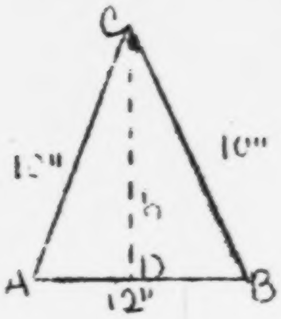
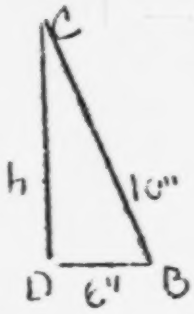


Lesson 16 (continued)

D. Classwork: Construct an isosceles triangle, legs 10", base 12".
   1. Draw triangle on graph paper.
   2. Construct altitude from C to AB.
   3. Measure altutude h.
   4. Calculate for h, using Pythagorean Theorem: $6^2 + h^2 = 10^2$
   5. Solve for h using trig. functions.
   6. Compare results of (3), (4), and (5).



Assignment: Problems on hectograph.
   1. Construct triangle and altitude.
   2. Measure h.
   3. Calculate h using Pythagorean theorem.
   4. Calculate h using trig. functions.



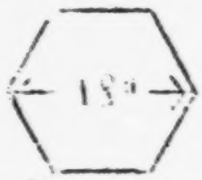
Lesson 17

I. Finding the area of a hexagon.

   A. Review homework.
      1. Compare results by three methods.
   B. Discuss the need to know both techniques of calculation.
      1. Indicate which given conditions make the Pythagorean theorem useful (two sides given).
      2. Which given conditions make the trig. functions useful (side and an angle given).

   C. Introduce application to area measure.
      1. Find area of hexagon with measurement across the corners of 18 inches.

         a. Draw figure, label given dimension.
         b. Analyze problem.
            i. Six equilateral triangles.



      2. Reproduce one of the six equilateral triangles.
         a. Construct an altitude.
            i. Base angles are 60°.
         b. Solve for h: $\sin 60° = \frac{h}{9}$.

Lesson 17 (continued)

c. Area of triangle by formulas: $A = \frac{1}{2}bh = \frac{1}{2}\,9h$

d. Area of hexagon is six times larger.

D. Classwork and assignment.

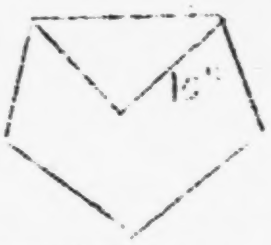
1. Slade and Margolis, pgs. 294, 295 #1-3
2. Find the area of a hexagon which can be inscribed in a circle with radius 6".

Lesson 18

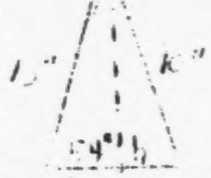
I. Finding area of a regular polygon, given the distance "across the corners".

A. Review homework.

B. Develop technique for finding area of a pentagon inscribed in a circle of radius 10".



1. Draw pentagon.
   a. Label parts given.
2. Find central angle: $\frac{360^\circ}{5} = 72^\circ$
3. Find base angle: $\frac{180^\circ - 72^\circ}{2} = 54^\circ$
4. Five isosceles triangles form the pentagon.
5. Reproduce one of the isosceles triangles.



6. Use sine function to calculate altitude of triangle.
7. Calculate area of triangle, multiply by five.

C. The same method can be used for any regular polygon when distance across the corners is given.

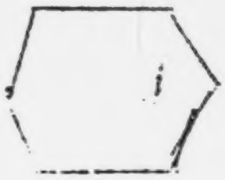
Assignment:
1. Find area of a pentagon inscribed in a circle of radius 12 inches.
2. Slade and Margolis: page 294, 295: #4, 5, 6.

Lesson 19

I. Finding the area of a regular polygon, given the distance "across the flats".

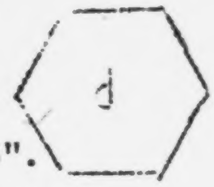
A. Review homework.

   1. Stress meaning of "across the corners" as the measure of the diameter of a circumscribed circle.



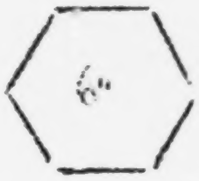
B. Introduce the concept of the "distance across the flats".

   1. Stress the meaning as the measure of the diameter of the inscribed circle.



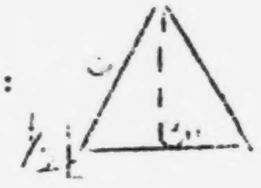
C. Develop the technique for finding the area of a hexagon if the distance across the flats is 6".

   1. Draw the hexagon and label parts.
   2. Draw one of the six equilateral triangles which can be fitted in the hexagon.
   3. Find the central angle: $\frac{360^\circ}{6} = 60^\circ$.
   4. Find one base angle: $\frac{180^\circ - 60^\circ}{2} = 60^\circ$



   5. Find half of base using cosecant function:

$$\csc 60 = \frac{c}{3}.$$



   6. Find the length of the base.
   7. Find the area of the triangle and multiply by xix.

D. Classwork and assignment.

   1. Find area of a hexagon if the distance across the flats is 8".
   2. Find the area of a pentagon if the distance across the flats is 8".
   *3. Find the area of a nonagon (9-sided regular polygon) if the distance across the flats is 10".

Lesson 20

I. Application of trigonometric functions to volumes of prisms.

A. Quiz: Find the altitude of an isosceles triangle with a vertex angle equal to 80° and base equal to 20".

B. Review quiz and homework

C. Introduce use of area concepts to find volume of right regular prisms.
   1. Volume = (area of base) x (height of prism).
   2. Review concept of fitting one layer of cubes on base, then determine the number of layers needed to fill the prism.

D. Classwork
   1. Find the volume of the following geometric solids, using trig. functions where applicable.
      a. Right pentagonal prism, base inscribed in a 2: radius circle and altitude of 4".
      b. Right octagonal prism, base inscribed in a 2" radius circle and altitude of 4".
      c. Review terms: "across the points" and "across the flats".

Assignment: Find the volume of a right pentagonal prism, base inscribed in a 3" radius circle and altitude 5".

Lesson 21

I. Angles of elevation and depression.

A. Review method of solving for area of any regular polygon.
   1. Given dimensions across corners.
   2. Given dimensions across the flats.
   3. Given radius of inscribed or circumscribed circles.

B. Introduce angle of elevation and angle of depression.

   1. Relate angle of elevation and angle of depression to parallel lines and a transversal.
   2. A film or film strip on the topic is helpful.

C. Classwork: Determining distances using angle of elevation or depression

Name ________________ Date __________

Assignment: Lesson 21

For each exercise, draw a figure, label all given parts, calculate the desired distance using the Pythagorean theorem or trigonometric functions.

1. A dam 15 feet high backs up the water in a lake for a horizontal distance of 300 feet. What is the average slope of the ground at the bottom of the lake from the base of the dam?

2. A railroad track rises 320 feet in one mile of track. What is the angle it makes with the horizontal?

3. A stay wire from the top of a telephone pole to the ground is 117 feet long. If the wire forms an angle with the ground of 43 20', how high is the pole?

4. An airplane is directly over a town, while an observer is 4 miles distant from the town. If the angle of elevation of the plane is 17 14' when sighted from the observer, how high is it?

5. A straight railroad track rises 200 feet in one mile of track. What is its inclination to the horizontal?

6. From a balloon, the angle of depression of a road intersection was 41 40'. If the balloon was 855 feet high, how long was the straight line from the balloon to the intersection?

7. A captive balloon is fastened by a cable 1,000 feet long. The balloon is blown by the wind so that the cable makes an angle of 63 50' with the ground. How high is the captive balloon?

8. The distance, AC, on level ground along a stream is 83 feet. Point B is on the opposite shore across from point C. Angle C in the triangle determined is 90 and angle CAB is 33 10'. How long is CB?

9. What is the angle of elevation of the sun when a monument 346 feet high casts a shadow 210 feet long?

10. The sides of a rectangular field are 890 yards and 540 yards. Find the angle formed by a diagonal and one shorter side of the field.

11. A lighthouse rises 289 feet above sea level. As observed from a ship, the angle of elevation of the top of the lighthouse is 9 26'. How far is the lighthouse from the ship?

12. An airplane is one mile from a tower in a horizontal distance. The navigator of the airplane sights the top of the tower and measures the angle of depression of the top of the tower as 28 35'. If the plane is 3,500 feet high, what is the height of the tower?

Assignment: Lesson 21 (continued)

13. A ladder leans against a house and stands on level ground. If the foot of the ladder is four feet from the house and the top of the ladder is eleven feet from the ground, find, to the nearest degree, the acute angle which the ladder makes with the ground. How long is the ladder?

14. In order to calculate the height of a mountain, a surveyor measured the angle of elevation of its top from point A and found it to be 17 18'. He then walked toward the mountain a distance of 3,200 feet in the same horizontal plane and found the elevation to be 22 6'. How high was the mountain above the plane?

15. What is the angle of elevation of the sun when a tree casts a shadow 1/3rd of its own height?

Lesson 21 (continued)

1. A man standing 120 feet from the foot of a tower finds that the angle of elevation of the top of the tower is 51.3°. Find the height of the chimney.

2. From the top of a building 160.2 feet high, the angle of depression of a car on a road is 26° 20'. How far is the car from the foot of the building?

Assignment: For each problem make a sketch and solve the problem.

1. At a horizontal distance of 112.0 feet from the base of a tower, the angle of elevation of the top is 72 10'. Find the height of the tower.

2. Find the angle of elevation of the sun when a tree whose height is 96 feet casts a shadow 116 feet in length.

3. What is the angle of elevation of an inclined plane if it rises a foot in a horizontal distance lf 12'?

4. The Washington Monument is 555 feet high. What is the angle of elevation of the top when viewed at a distance of helf a mile?

Lesson 22

I. Finding angles of elevation and depression.

A. Review homework: illustrations and problems on board.

B. Classwork. Finding angle of elevation and depression.

1. A garage is made using gable rafters 10 feet long, with a pitch of 20°. The rafters project one foot beyond the walls of the garage. Find the height of the ridgepole and the width of the garage.

Assignment: (hectographed page.) Draw the figure for each problem, lable all given parts. Do #2, 3, 4, 5.

Lesson 23

I. Introduction to the concept of slope.

  A. Slope is the ratio of the "rise to the run".
    1. Recognize this ratio as the tangent ratio.
    2. Illustrate with sets of stairs.
      a. Give height between floors, horizontal length across stairwell opening.
      b. Show use of equivalent fractions and decimal value to tangent ratio.
  B. Classwork: Hetographed page: #1, 2.
    Find the slope of the ground in each problem.

Assignment: Hectographed page: #6, 7, 8, 10, 13.
  Find the slope of the oblique line in each problem.

Lesson 24

I. Application of trigonometric functions to slopes.

  A. Quiz: Find the angle of elevation of a 40' long ladder against a building if the foot of the ladder is 6 feet from the building. Draw figure showing the situation.

  B. Review quiz and homework.

    1. Draw illustration of each problem, analyze, and solve.

  C. Classwork and assignment: Hectographed sheet.
    Complete #10, 11, 13, 14, 15, 16.

Lesson 25

I. Application of trigonometric functions.

  A. Quiz: Find the angles of an isosceles triangle if the equal sides are each 12 inches and the base is 18 inches.

  B. Review homework.

  C. Classwork and assignment. New hectographed sheet, #1-7.

Name____________________ Date ____________________

Assignment: Lesson 25

1. Which of the following triples of numbers could be the lengths of the sides of a right triangle?

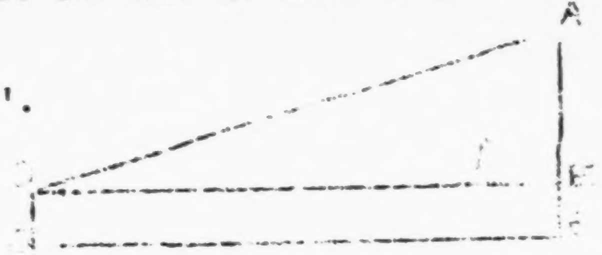
a) 10, 24, 26
b) 8, 14, 17
c) 7, 24, 25
d) 9, 40, 41

2. Find the length of a diagonal of a square whose sides have length twelve units.

3. When propped against the side of a house, a ladder 18 feet long just reaches a window sill. If the window sill is 15 feet above the ground, how far from the side of the house is the foot of the ladder. Find the angle of elevation formed by the ladder with level ground.

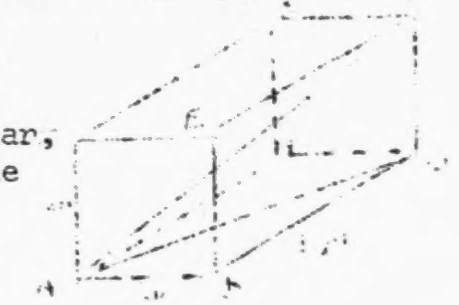
4. Two cars leave a town at the same time. One travels north at an average rate of 30 m.p.h. and the other travels west at the average rate of 40 m.p.h. How far apart are the cars at the end of 1.5 hours?

5. IN the figure at the right:
AB = 38', BC = 60', and CD = 2'.
Find the length of AD.

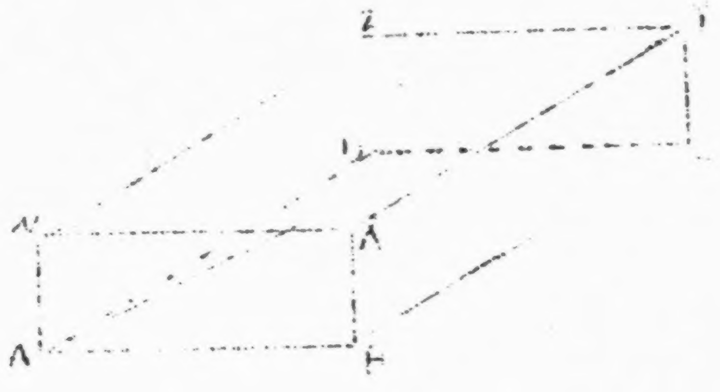


6. The lengths of the legs of right triangle ABC are 15' and 8'. Find:
a) The length of the hypotenuse,
b) The length of the altitude on the hypotenuse.

7. Refer to the drawing at the right. Given a rectangular prism in which any two intersecting lines are perpendicular, find the lengths of AC and AD using the given information.



8. Refer to the figure at the right. In the right rectangular prism, find the length of AY if AB = BC = 2' and AW = 1'



Lesson 26

I. Review of trigonometric functions.

   A. Review homework.

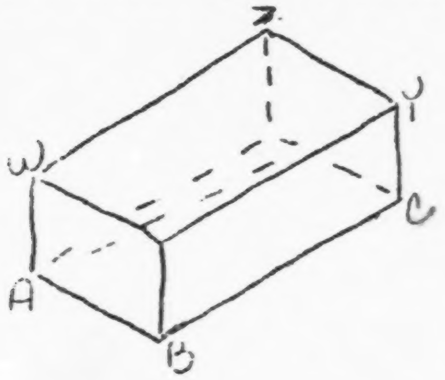
      1. Include question and answer period for test review.

Assignment: Study for test.

Lesson 27

I. Test.

   A. Include for extra credit: In the rectangular prism illustrated find AY if AB = BC = 2, and AW = 1.



Lesson 28

I. Introduction to properties of materials.

   A. Review test.

   B. Basic materials from metal and wood shops.

      1. Models of iron, aluminum, brass, copper, wood, zinc, plastic.

         a. Select one set of models having the same cross section and length.

      2. Discuss the characteristics of the materials.

         a. Metal - wood - plastic.
            i. Different color.
            ii. Different feeling of softness, hardness, cold, warm.
            iii. Different weight.

Lesson 28 (continued)

- C. Discuss forms of each of the above materials.
    - 1. Wood
        - a. Trees, boards, furniture, plywood, paper.
    - 2. Metals.
        - a. Sheet, bar stock, beams.
- D. Uses of materials.
    - 1. Varied, interchangable.
        - a. Metal has replaced wood.
        - b. Corrugated (paper) board has replaced wood.
        - c. Plastices have replaced metal, wood, paper, glass.
        - d. Industry looks for usage of materials for new applications.
        - e. Factors involved with choice of materials.
            - i. Strength, resistance to corrosion, cost, weight, etc.
- D. Weights of iron, brass, copper, aluminum.
    - 1. Weights of each given in tables for one cubic foot.

Assignment: Given one cubic foot of iron in the form of a cube,
1. How many cubic inches are there in the cubic foot?
2. If iron weights 480 lbs./cubic foot. Find the weight in pounds of one cubic inch.

Show all calculations and pictorial views of the objects described.

Lesson 29

- I. Introduction to concept of weight and density.
    - A. Review homework.
        - 1. 1-1 correspondence between one cubic foot and cubic inches.
        - 2. Conversion from lbs./ cubic foot to lbs./cubic inch.
    - B. Weights.
        - 1. Meaning of weight.
            - a. Use two boys, one light and another heavy.
            - b. Comparison by lifting each off the floor.
            - c. Student interpretation of weight.
            - d. Definition of weight.

Lesson 29 (continued)

2. Weights and measures.
    a. Standard.
        i. English, metric.

3. Using pound as a unit of measure.
    a. Refer to previous topics for meaning of a unit of measure.

4. Some symbols for English system of weights.
    a. Pound (lb.)
    b. Ounce (oz.): 16 ounces = 1 lb.
    c. Ton.
        i. Long ton 2,200 lbs.
        ii. Short ton 2,000 lbs.

C. Weights of materials.

1. Refer to Table V, Slade amd Margolis, page 564.
    a. Compare weights of:
    iron, aluminum, brass, copper, white pine.
    b. According to Table V,
        i. What material weighs most?
        ii. What materials has the least weight?

Assignment: Display a piece of bar stock. Students take necessary measurements to:

1. Compute the base area (cross sectional area).
2. Compute surface area (total).
3. Compute volume.
4. Determien weight to nearest hundredth of a pound.
5. Determine weight to the nearest ounce.

Lesson 30

I. Calculation, using density, to find weight of an object.

A. Review homework.
1. Inform class of need to review area and volume of geometric forms.
2. Student review of homework problems at the board.
    a. Isometric view of prism.
    b. Compute volume and weight of prism.
        i. If volume is in cubic inches, change density to lbs./cu. inch.
        ii. Discuss simplicity of problem when corresponding units of measure are used.

Lesson 30 (continued)

- B. Classwork and homework.
  - 1. Compute weight of the following objects:
    - a. Hexagonal iron bar, 1" across the corners, 4" long.
      - i. Assume the density of iron is 480 lbs./cu. ft.
    - b. Flat iron stock with 1" x 1/4" base, 5" long.
      - i. Assume the density of iron is 480 lbs./cu. ft.

Lesson 31

I. Calculating to find weight, given density.

- A. Review homework.
  - 1. Develop format for problem solving.
    - a. Order the steps in find a soulution.
  - 2. Students put homework problems on board.
    - a. Isometric view included.
    - b. Use of formulas for volume included.
- B. Classwork and assignment.
  - 1. Display model of square aluminum bar stock.
    - a. Dimensions: 3/4" x 3/4" cross section, 5 1/2" long.
  - 2. Find density of aluminum in tables.
  - 3. Compute weight of stock.
  - 4. Display model of flat iron stock.
    - a. Dimensions: cross section 4" x 1/2", 6" long.
    - b. Complete drawings and calculations.

Lesson 32

I. Calculations for weight

- A. Review homework.
  - 1. Two students at board for each problem.
    - a. One for drawing and other for computation.
- B. Classwork and homework
  - 1. Find the surface area and weight of each object.
    - a. 1/2" round stock 8" long, made of iron.
    - b. 2" x 1/2" flat stock 3" long made of copper.

Lesson 33

I. Weight of materials.

  A. Review homework.
    1. Stress neatness, procedure in solving problem.

  B. Classwork and homework.
    1. Find weight of each of the following:
      a. Octagonal stock: 1" across the corners, 6" long, made of brass.
      b. Octagonal stock: 1" across the flats, 6" long, made of aluminum.

Lesson 34

I. Introduction to determining length of bar stock having a required weight.

  A. Develop the technique for finding a length of required weight.
    1. Volume = (area of cross section) x length of prism.
    2. Weight (lbs.) = volume (cu. ft.) x density (lbs./cu. ft.)
    3. Thus:
       Weight = (area of cross section) x length x density.
    4. Make all substitutions, simplify calculations, then solve for length using division axiom.

  B. Classwork and assignment:
    1. Find the length of bar stock to give the required weight.
      a. Hexagonal iron stock, 1/2" across the corners.
      b. Round aluminum stock, 1" diameter.
      c. Square iron stock, 1" on each edge.
      d. Rectangular iron stock: L" x 1/2" in cross section.

Lesson 35

I. Determining length of bar stock having a required weight.

  A. Review homework.
    1. Stress technique of substitution and using the division axiom to solve for length.

Lesson 35 (continued)

B. Classwork: Find the length of bar stock to give the required weight.
    1. Hexagonal iron stock 1/2" across the flats.
    2. Round iron stock 3/4" in diameter.
    3. Square iron stock 3/4" on each edge.
    4. Rectangular aluminum stock 3/4" x 1 1/2" in cross section.

Lessons 36 - 38

I. Introduction to individual projects on determining the length of stock having a required weight.

  A. Each student is assigned three different types and sizes of bar stock and:
      1. Each must calculate the length of stock necessary to yield a piece of required weight.
      2. Each must draw orthographic projection and an isometric drawing of the required piece of bar stock.
      3. Each student must go the metal shop, cut off length of each bar stock required.
      4. Each student must weight pieces of bar stock.
          a. In science room.
      5. Each student compares actual weight to desired weight.

Lesson 39

I. Test on use of trigonometry in determining the volume and weight of prisms.

UNIT4

MATHEMAMATICS OF PACKAGING

Project 4 Packaging

Mathematics topic: Paper box design and fabrication.

Lesson 1.

I. Introduction to paper box design and fabrication.

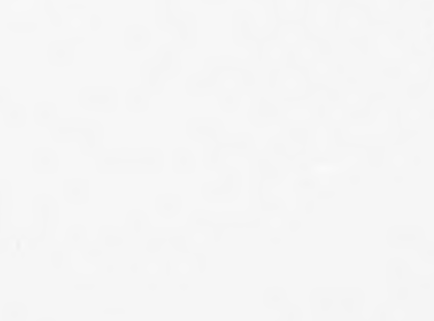
  A. Universal use of paper packages, cartons, and paper boxes throughout this country.
    1. Demonstrate samples, indicating the variety of companies, variety of shapes and sizes.
    2. Open discussion with students concerning the varied uses of paper cartons.
    3. Indicate the size of this industry and its relation to other industries.

  B. Indicate the type of materials used and their functions:
    1. Boxboard.
      a. For containing small items with advertising.
      b. For containing hats, suits, cakes.
      c. Note the qualityof printing possible.
    2. Corrugated board.
      a. For containing larger, heavier items.
        1. For protection.
      b. For convenience in shipping many items.
      c. For convenience in stacking, inventory, and retrieval.
      d. Show the corrugated sleeve which is used to contain a light bulb. Discuss the purpose of the corrugation.
      e. Discuss the type of printing on corrugated.
      f. Display a carton made with 1/16th thick corrugated.
    3. Blister packs
      a. For display, advertising, protection, to avoid thievery.
    4. Styrofoam
      a. For protective inserts.
        I Preformed.
        II Solution poured in box, expands around contents.
    5. New products, new developments continually being sought.

  C. Outline the plans for this unit of study.
    1. We shall study the application of measurement required for the design and fabrication of the basic types of patterns developed by the paper box industry.
    2. We shall analyze and study the dimensioning of patterns using actual commercially-made patterns.

3. Related to dimensioning, we shall study the use of tolerance of measurement used in folding-box designs. This will apply to our work with corrugated board.
4. We shall construct cartons from folding box board and corrugated board.
    a. Supplied a local manufacturer.
    b. We shall use hand-scoring tools, as used by designers.
5. We shall expect groups of students to work together on developing cooperative projects in packaging.

II. Introductory Project.
  A. Students are to learn the basic concepts involved in fabricating a paper box by applying their knowledge from earlier work.
    1. The students should, upon completion of the project, realize the inadequacy of their knowledge when applied to this new study.
  B. Classwork assignment:
    1. Students design and fabricate a paper box from folding boxboard to hold a wooden dowel (cylinder or jar) with diameter and height given. (Suggest d=2", h=5".)
        a. Introduce pattern design for a paper box as "a length of square duct with ends attached", from folding box board.
            i. Students will tend to make flaps too narrow.
            ii. Call on students who may have investigated package design on their own.
        b. Teacher purposely avoids teaching special techniques.
        c. Teacher and students discuss and sketch a pattern for the box including all flaps.
            i. Top and bottom are purposely attached to same lateral face.
            ii. All lines for faces are drawn in only two directions (perpendicular).
            iii. Note: The terms length, width, and depth are always given in that order.



Lesson 1 (cont'd)

        d. Remind students to score all lines.
            i. Use handle of spoon or coin.
            ii. Score the lines to be cut also.

Assignment: Using regular instruments: T-square and triangle is possible, draw pattern for a box to hold a dowel with base diameter 2" and depth 5". Score and cut out.

1. Find the area of the pattern by finding area of parts. Treat flaps as though they were rectangles.
2. Find area of smallest rectangle from which pattern can be cut.
3. Find the surface area of the box. Find volume of the box.
4. Find the volume of the cylinder and the volume of the space remaining inside the box.

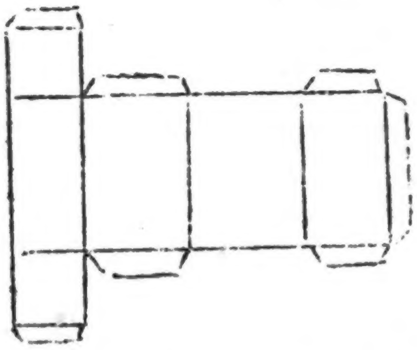
Lesson 2

I. Introduction to paper box design.
    A. Review homework.
        1. All calculations and related sketches on the board.
            a. Compare area of pattern and total area of box.
            b. Compare volumes of cylinder and box.
        2. Students glue side and bottom flaps. Check measurements.
            a. Teacher checks by setting several boxes on desk top and compares heights.
                i. Teacher and students can readily spot dimensions which are "off".
        3. Teacher and students observe and analyze problems in their design and fabrication.
            a. Do all folds have a sharp radius bend?
                i. Teacher reviews use of scoring die and hand tool for creating a sharp radius bend.
            b. Are all faces flat? Do top and bottom fit flush?
            c. Does box wobble when face is placed on desk top?
                i. Edges are not parallel, opposite edges are not parallel.
                ii. Adjacent edges not perpendicular.
            d. Does the cylinder fit snugly in the box without bulging?

Lesson 2 (continued)

e. Would the bottom, unglued, hold the wooden cylinder?
   i. Let it drop through once to demonstrate the need for "locks" on the covers of a box.

f. Note that the sides of the box are rigid, but the top and bottom can be easily bent in before the box is glued together.
   i. Show the strength and rigidity of paper when creased and folded.
   ii. Stress the need for a flap at top and bottom of the box to "tuck" in, as found commercially-made boxes.

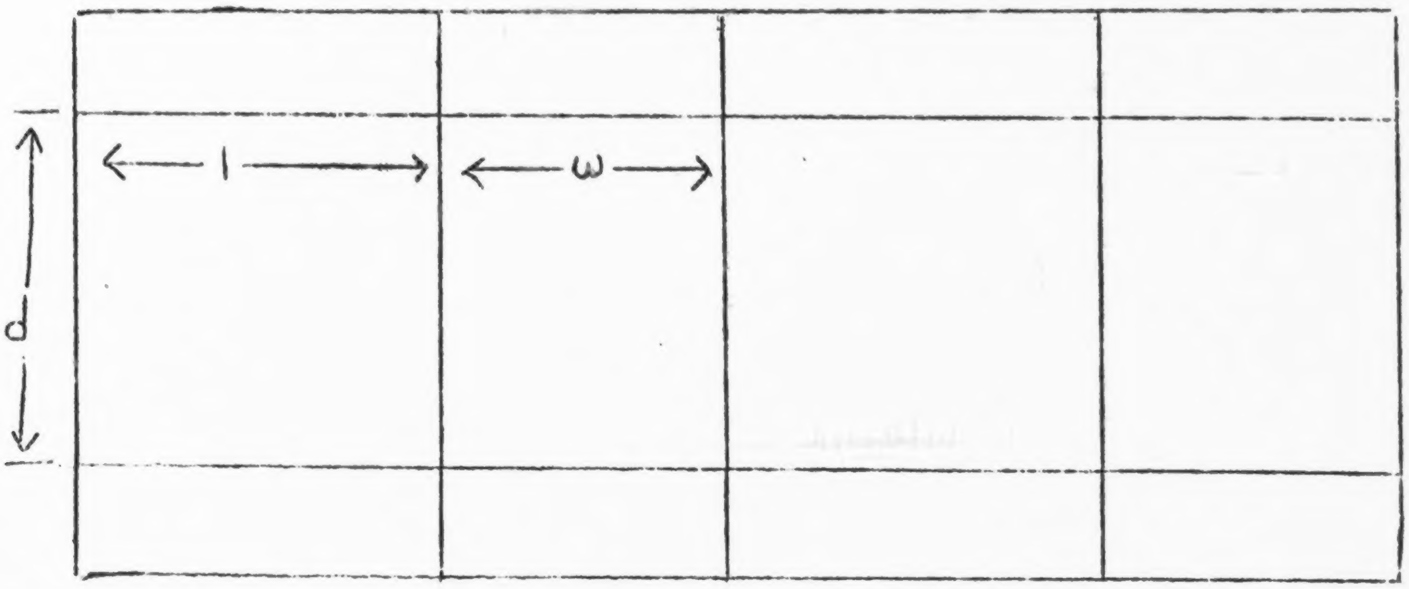


Assignment: If the first pattern was not satisfactory, construct a second pattern to fit the same object.

1. Pattern should have tuck flaps at ends.
2. Find the area of the smallest rectangle from which the pattern can be cut.
3. Find a way to fit copies of this pattern on a rectangular sheet of paper 18" by 20". Make a sketch.
4. Extra credit. Sketch four (4) other patterns for the same box and determine the dimensions of the smallest rectangle from which the pattern can be cut.
5. Fabricate one of the patterns in #4.

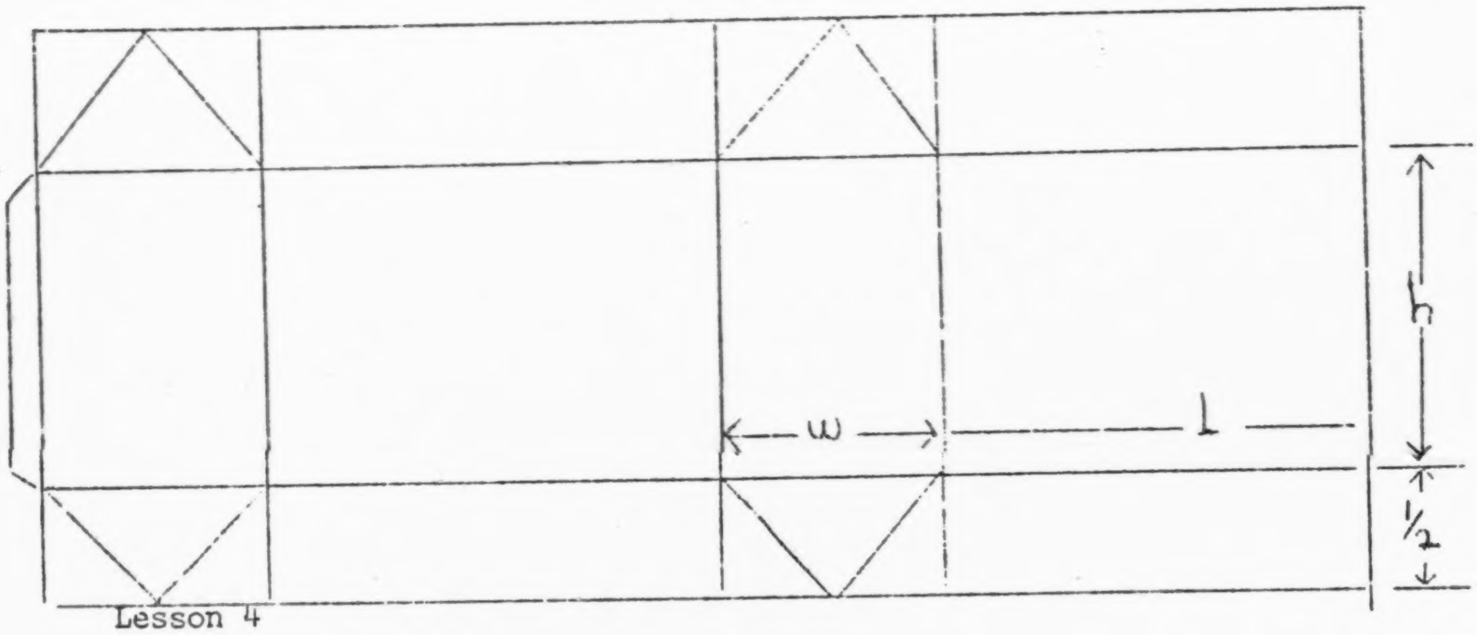
Lesson 3

I. Introduction to first pattern: The Regular Slotted Carton.
  A. Review homework
    1. Sketches on board showing how to fit patterns on 18" x 20" rectangle. Students can demonstrate with an actual sheet.
    2. Sketches on board showing variety of patterns for the same box.
      a. Show overall dimensions.
    3. Students complete gluing second pattern. Check by comparison with other boxes.
  B. Students and teacher examine a commerical sample of a Regular Slotted Carton, made of corrugated board.
    1. Note use of large flaps on each end of side panels.
      a. For rigidity and strength, and to keep box square.
    2. All edges are straight lines. All lines go in either of two perpendicular directions.
      a. Describe that these box patterns are made in two operations, passing the pattern through rollers and then through a die board.
    3. Do not consider the type of scores or allowances for folds.
  C. Pass out boxboard (not corrugated).
    1. Students assigned to fabricate a box to hold a object using the Regular Slotted Carton design.
      a. Students receive a copy of the pattern.
      b. Give dimensions of the object.
  D. Show students a variation of a regular Slotted Carton used for milk containers.



Lesson 3 (cont'd)

Extra Credit. Design and construct a pattern for the variation of the RSC shown in the lesson. Use your own measurements. Determine how to fold and fabricate this pattern.



Lesson 4

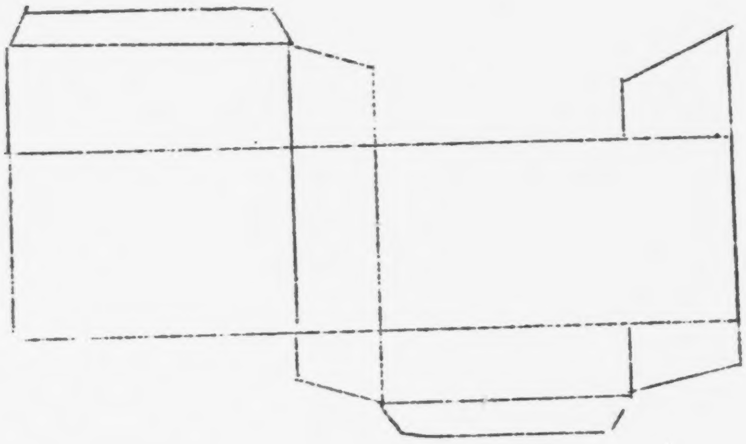
I. Introduction to the Reverse Tuck pattern.
  A. Review homework.
    1. Glue boxes together at sides only.
    2. Calculations and related sketches on board.
      a. Show comparisons in area of pattern and box.
      b. Show comparisons in volume of cylinder and box.
    3. Display designs of variation of pattern for RSC.
      a. Discuss the amounts of material needed for the different designs.
  B. Teacher demonstrates samples of Reverse Tuck Box pattern.
    1. Students make observations.
      a. Reverse tuck has flap or top on all but two edges.
      b. Dust flaps (at top and bottom of box) are not rectangular, as on the slotted carton.
        i. Reason? To keep dirt and dust out.
        ii. Demonstrate idea of a lock later.
      c. Top and bottom faces are attached to opposite sides of the box, causing top and bottom to be "tucked in" in opposite, or reverse, directions.
  C. Reasons for wide spread use of the Reverse Tuck design.
    1. Rigidity of sides, due to folds and flaps.

Lesson 4 (cont'd)

2. Economy of material in fabrication.
    a. Demonstrate the placement of several copies of a Reverse Tuck pattern on a large sheet of paper.
    b. If possible, show dieboard or picture of a dieboard for Reverse Tuck patterns.
3. Discuss the design of locks on the dust flaps.
    a. Offset of flaps at corner to put pressure on the tuck flaps.
    b. Measurement must be carefully made.

Classwork and assignment:

1. Students design and fabricate a reverse tuck box to hold a cylinder with diameter 2" and height 5".
2. Find the area of the pattern, surface area of the box.
3. Find the number of patterns which could be cut from a rectangular sheet of paper 18" x 20".
4. Compare results to those for first pattern of Lesson 1.



Lesson 5

I. Reverse Tuck design.
    A. Quiz.
        1. Teacher gives isometric drawing of box and its dimensions. Students are to make a sketch of a Reverse Tuck pattern, indicating dimensions. Or students dimension a pattern supplied by teacher.
    B. Students analyze projects from homework.
        1. Students glue only side gluing flaps. Ends tuck in.
        2. Compare dimensions of boxes by matching.
        3. Area and volume sketches and computations on board.
            a. Compare area of pattern to that of earlier pattern.

- b. Compare number of patterns which can be cut from an 18" x 20" sheet.
- c. Discuss overall cost and possible loss.

4. Teacher demonstrates the use of flaps, locks, and their design.
   - a. Place a brass or heavy dowel of given dimensions in a box which was not well fabricated. (Watch your toes)
   - b. Dramatically stress the need for flaps and locks which fit snugly enough to hold a heavy object.
   - c. Demonstrate and discuss various lock designs.
     - i. Slit on flap at top. Butter cartons (1 lb.)
     - ii. Offset on flaps.
     - iii. Mailing flaps.
   - d. Note different types of locks used on bottoms of boxes.
     - i. For convenience in opening the correct end.

C. Assign students to measure a flask from the Science Department and form a pattern for a Reverse Tuck box pattern to hold the flask securely.
   1. Find the area of the pattern.
   2. Find the area of the box.
   3. Find the volume of the box.

Extra credit. Estimate the volume of the flask. Base your technique on volume of a cone with the same base.

Lesson 6

I. Interliners to protect contents.
    A. Review homework.
        1. Calculations and sketches on board.
        2. Some students demonstrate the number of patterns which can be fitted on a rectangular sheet 18" x 20".
        3. Students glue side flaps only. Close top and bottom.
        4. Check measurements of boxes by matching a set of them.
        5. Stress the importance of obtaining the greatest possible number of patterns from each sheet.
    B. Discuss the use and design of protective inserts.
        1. To keep neck of bottle from rattling.
        2. To cushion the bottom of the bottle.
    C. Discuss the design of a box to include cushioning and protective inserts.
        1. Students begin planning, sketching patterns.
        2. Discuss allowances which must be made to have inserts fit.

Assignment: Plan and sketch a pattern of a Reverse Tuck design to hold a flask, including protective cushion and a protective top. Use the same flask as used for today.
1. Find the area of the pattern. Do not account for holes in the protective insert.
2. Find the surface area of the box.
3. Find the volume of the box.
4. Using the volume of the flask determined today, find the volume of the space not used (outside the flask.)

Extra credit: Design a pattern for a box and all inserts from one piece of paper. Do this in place of the required pattern if you think you have a workable idea.

Lesson 7

I. Quiz.
    A. Give students a hectographed page displaying the layout for a Reverse Tuck pattern. Give the dimensions of the box.
        1. Students are to write in all dimensions for the pattern, if it is to be made from folding box board.
        2. Find the volume of the box and the surface area of the box.
    B. Review quiz.

II. Review homework assignment.
    A. Calculations and sketches on the board.
        1. Compare areas and volumes of this carton with those of the first carton for the flask.

Lesson 7 (cont'd)

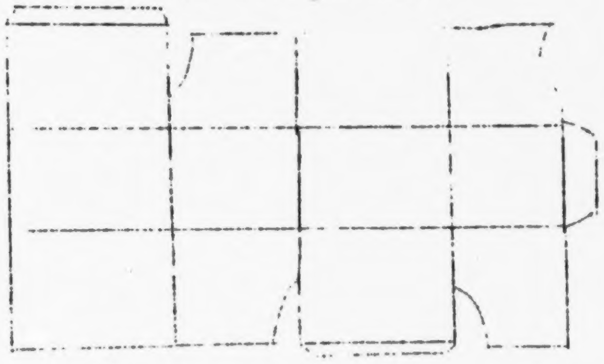
- B. Students display samples of one-piece patterns.
- C. Students glue side flaps only, place inserts and test function of box with flask.
  - 1. Discuss the different methods of cutout for protective inserts.
    - a. Circular hole with slots radiating from edges.
    - b. Perpendicular slots so that corners can be pushed into the neck of the flask.

III. Introduce the pattern for the "Tuck Top with 996 Bottom" design.

- A. Purpose: to create an especially strong bottom with a strong lock.
  - 1. Students receive a mimeographed drawing of the pattern.
- B. Challenge students to devise a method for fabricating the pattern.
  - 1. Glue side flap first. All other panels fit together if correct steps are used.
  - 2. Present a commercially-made pattern and challenge students to follow the sequence of steps which they developed.

Assignment: Construct a second pattern for a Reverse Tuck box with protective inserts.

Extra credit: Design and fabricate a Tuck Top with a 996 Bottom to hold a cylinder having a base diameter of 2" and height of 5". Use 5" as length and 2" as depth. Find the area of the pattern.



Lesson 8

I. Introduction to estimation of material.
  A. Review homework assignment.
    1. Students glue side gluing flap only.
    2. Teacher compares boxes for dimensions by matching.
    3. Students demonstrate their designs for inserts.
    4. Extra credit projects demonstrated.
  B. Teacher poses problem encountered in the paper box industry.
    1. How many sheets of folding paper board will be required for a given job?
      a. Refer to problems from earlier homework assignments.
      b. Name the various patterns and the number of each which can be cut from an 18" x 20" sheet.
    2. Present one type of problem.
      a. If we can cut x patterns from each sheet, how many patterns can be cut from y sheets? (x and y are integers.)
  C. Classwork: Give each student either a set of commercially-made patterns or set of hectographed copies of various sizes and shapes.
    1. Students are to determine the number of each pattern which can be cut from an 18" x 20" sheet.
    2. Some students should trace each type of pattern on 18" x 20" paper.

Assignment: Determine the number of each pattern which can be cut from each of the following numbers of sheets of 18" x 20" paper: 2, 5, 10, 50, 100, 1,000

Pattern from Lesson #	Number of 18" x 20" sheets				
	2	5	10	50	100
1					
2					
3					
4					
*5					

Lesson 9

I. Introduction to estimation for materials needed.

- A. Review homework calculations and sketches on board.
  - 1. Some students demonstrate their layout of patterns on 18" x 20" paper showing the maximum number obtained.

- B. Introduce problem of estimation used in industry.
  - 1. If we are required to cut m patterns and we can cut x patterns from each sheet, how many sheets will be required?
    - a. Discuss feasibility of selecting larger or smaller sheets of paper to minimize waste.
    - b. Discuss feasibility of cutting other patterns from the same sheet to minimize waste.

- C. Classwork: Find the number of sheets required for each given number and type of pattern.
  - 1. Use the patterns from the homework assignment.
  - 2. For each pattern determine the number of sheets needed to produce each of the following number of sheets:
    - a. 30, 200, 1,000, 15,000, 40,000
  - 3. Refer to these problems. If each sheet costs $.05, find the cost of cutting each number of patterns.
    - a. Relate this concept to other industries.
      - i. Wood, metal, plastics, clothing.

Assignment: Refer to the patterns used in last night's assignment. Determine the number of sheets 18" x 20" to cut the following number of patterns: 100, 5,000, 25,000, and 35,000.
Find the cost of paper if each sheet costs $.04.

Pattern for Lesson #		Number of patterns			
	30	200	1,000	15,000	40,000
1					
2					
3					
4					
*5					

Lesson 10

I. Half-period test.

   A. How many patterns of a Regular Slotted carton with l=3", w=2", and d=4" can be cut from boxboard measuring 16" x 21"?

   B. How many patterns can be cut from 400 sheets of boxboard if 15 patterns fit on each sheet of paper?

   C. We can fit twelve patterns on each sheet of paper. How many sheets will be needed to cut at least 23,000 patterns?

II. Review test and homework.

   A. Calculations on board.

III. Introduction to the use of corrugated board in paper box fabrication.

   A. Discuss special uses of corrugated board.
      1. Protection in transit, ease of stacking (storing).
      2. Savings in cost compared to wood or metal.
      3. For transporting large and/or heavy items.
      4. When waterproofed, for vegetables, frozen foods, meats.
      5. Reusable boxes, due to chemical treatment.
      6. For cushioning or protection when made in layers.
      7. For displays in stores
         a. Show a sample
      8. Corrugated paper is used to hold light bulbs.
         a. Discuss reason for direction of fluting.

   B. Introduce problem of measurement with corrugated board.
      1. Each student receives two strips of corrugated board approximately 11" by 3" from B flute (1/8" thick),
         a. Fluting runs lengthwise on one piece, along width on other.

Assignment: Score each piece in such a manner that it will form a sleve which will fit snugly, without bulging or binding, on a 2" x 4" block of wood. Actual dimensions are 1 3/4" by 3 3/4".

Note: Teacher may need extra strips for those who make errors.

Lesson 11

I. Introduction to the use of corrugated board in paper box fabrication.

- A. Review homework: Teacher anticipates that students did not make allowances for losses in folding corrugated board.
    - 1. Teacher seeks students who were able to make corrugated board fold and fit correctly.
        - a. Discuss why some students failed.
            - i. Lack of effort and thought.
            - ii. Did not allow for shrinking of material when folded.
    - 2. Teacher keeps first attempts for a demonstration of strength of the material in later lessons.

- B. Teacher presents composition and characteristics of corrugated board.
    - 1. Construction: fluting, liners, glue.
        - a. Technique of making fluting at the mill.
    - 2. Kraft paper: classified by weight per 1,000 sq. ft.
        - a. Show samples.
    - 3. Classification of corrugated board.

Type	Thickness	Number of Corrugations per foot.
A	3/16"	36
B	1/8"	48-52
C	5/32"	42
E	1/16"	96

- a. Show examples of these as used in commercially-made cartons.

- C. Students are asked to re-measure the dimensions on each piece of B flute.
    - 1. Next fold the board at one fold and measure each panel which has the fold line as an edge.
    - 2. Note that each panel loses 1/16" when folded.
        - a. Where does the 1/16" go? Into the fold.
    - 3. Measure outside dimension of each panel which has the fold as an edge.
        - a. Note the panel gains approximately 1/16" when folded.
- D. Students should observe these characteristics of B flute regardless of the direction of the fold.
    - 1. Inside panel loses 1/16" at each fold.
    - 2. Outside of panel gains 1/16" at each fold.

Lesson 11 (cont.)

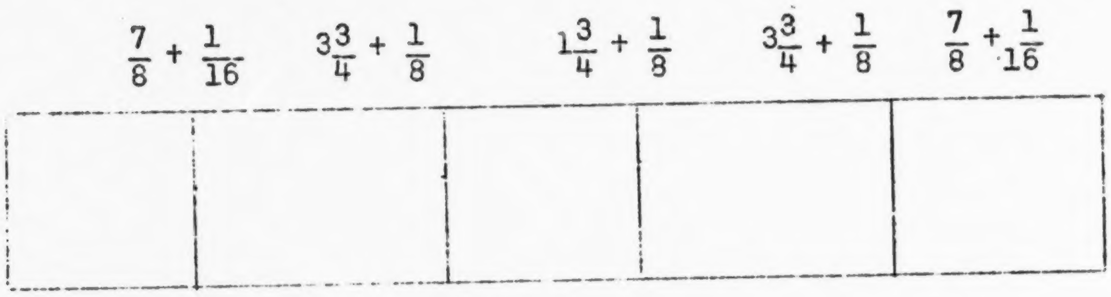
E. Classwork and assignment:
   1. Using pieces of B flute 3" wide and about 12" long measure, score, and fold the board to form a sleeve which will fit around a 2" x 4" board (actual dimensions 1 3/4" x 3 3/4").
   2. Make a sketch of the sleeve showing the inside dimensions and outside dimensions of the sleeve when it is folded.
   3. Make the sleeve so that the ends but-join on one face of the block of wood.
   4. Find the inside area of the sleeve. Develop an easy way to find this?
   5. Find the outside area of the sleeve. Develop an easy way to find this?

Lesson 12

I. Continue study of allowances for folding corrugated board.

   A. Students present sleeves and discuss problems.
      1. Test sleeves by fitting them to a 2 x 4 block.

   B. Students show sketches and dimensions of pattern.
      1. Show calculations for area.
         a. Quick way is to find total area of pattern and area of part lost due to folding and subtract.
      2. Dimensions of pattern.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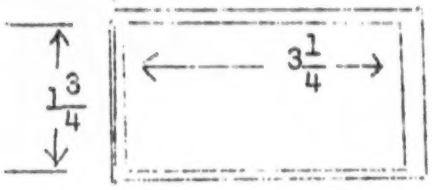
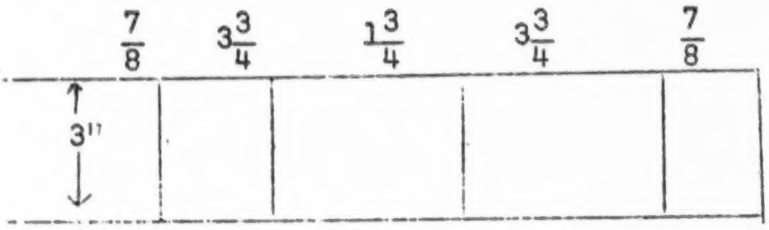
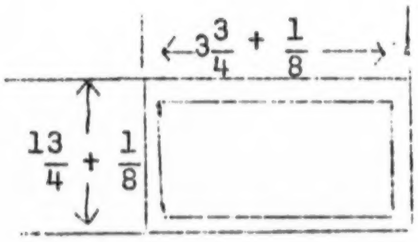
      3. Dimensions of inside of sleeve.

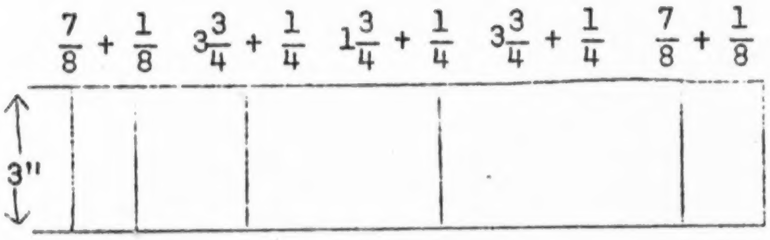
Lesson 12 (cont.)





4. Dimensions of outside of sleeve.

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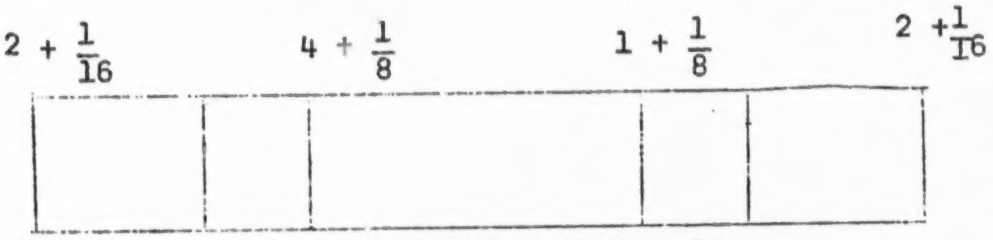
$3\frac{3}{4} + \frac{1}{4}$
$1\frac{3}{4} + \frac{1}{4}$
$3\frac{3}{4} + \frac{1}{4}$
$\frac{7}{8} + \frac{1}{8}$
3"

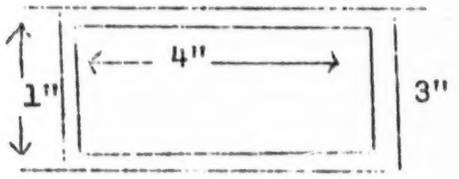
5. Again calculate inside area and outside area by finding the total length of each and multiply by 3.
   a. Students will need help with addition and multiplication of fractions.
   b. Compare results with other methods.
   c. Discuss the fact that the total outside area of the sleeve is greater than the total inside area.

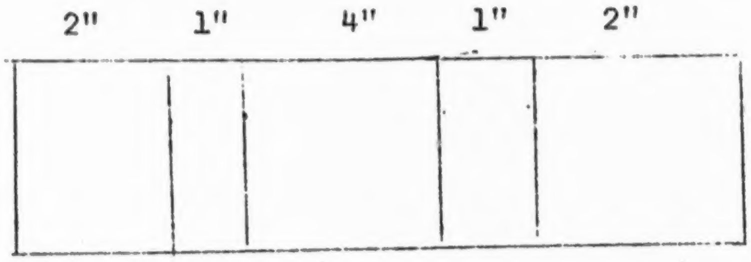
C. Classwork: Show sketches and dimensions for a pattern of sleeve to fit around a block 1" x 4" (actual). Calculate the area inside and outside.

1. Dimensions of the pattern.



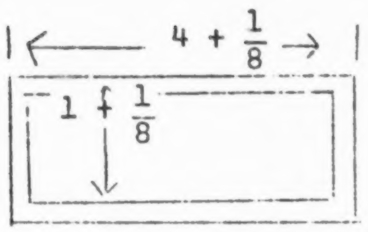
2. Dimensions of the inside of the sleeve.

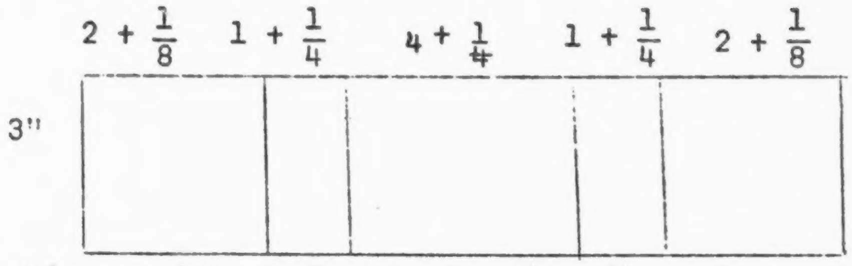




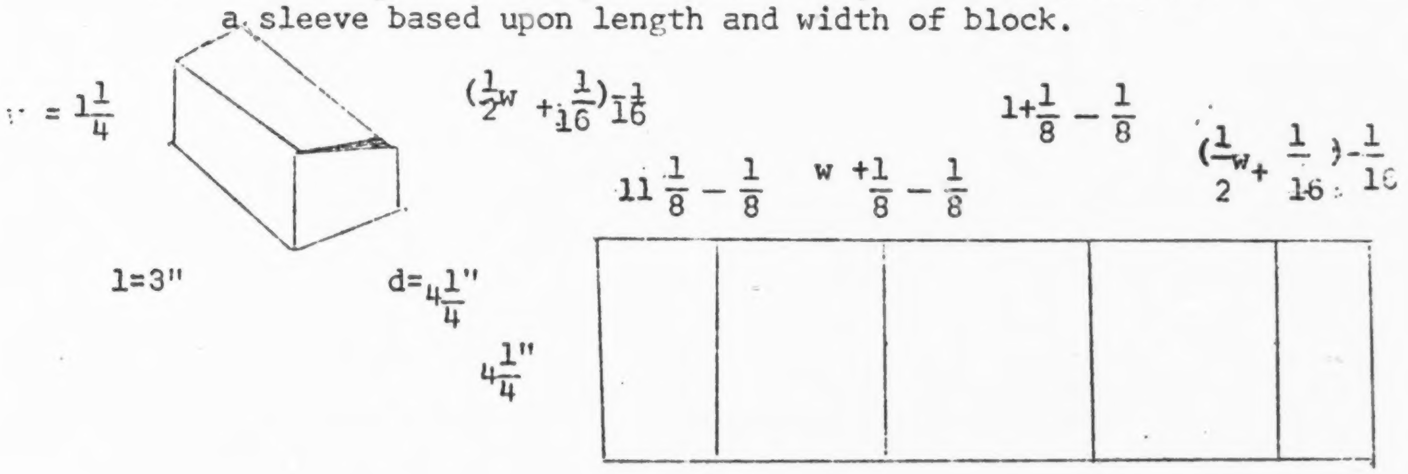
Lesson 12 (cont.)

3. Dimensions of the outside of the sleeve.





D. Teacher generalizes, with students, the dimensions for a sleeve based upon length and width of block.



Assignment: Teacher demonstrates a sample of a book mailing package made as two sleeves, each of which butt-join on a face, folding at right angles to each other so that all edges of the book are covered.

Make sketches of an outer pattern for the sleeve made in class to fit a block of wood 1" x 4" x 3".

Draw, score, cut out, and fold the outer sleeve from B flute.

Find the inside area and outside area of each sleeve.

Find the area of each pattern.

Lesson 13

I. Making allowances for folding corrugated board.
    A. Quiz.
        1. Sketch a pattern and show the dimensions for each panel to fit a sleeve around a block 2" x 3" with a butt joint on one face of the block.
            a. Make a sketch showing the inside dimensions of the sleeve.
            b. Make a sketch showing the outside dimensions of the sleeve.

    B. Review quiz.
    C. Review homework.
        1. Sketches and calculations on board.
        2. Students demonstrate the fitting of one sleeve around the other.

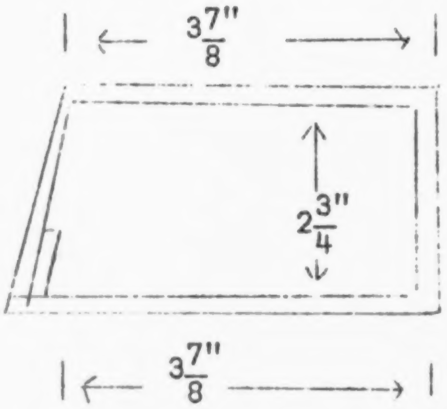
    D. Teacher can demonstrate the strength of corrugated board.
        1. Select four sleeves of B flute (Lesson 11) having the fluting in the short direction. Place them, nested, on the floor with the long edge down and folded as sleeves.
            a. They should hold the teacher's weight.
        2. Select four sleeves having the fluting in the long direction. Nest them and place them on the floor with the long edges down.
            b. The teacher's weight should easily crush them.
        3. Conclusion: corrugated board has great compression strength in the direction of the fluting.

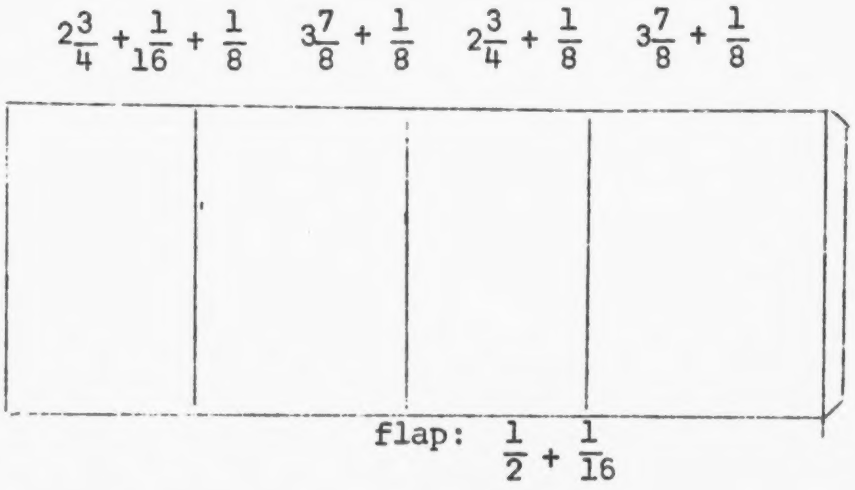
Assignment: Teacher challenges students to design a sleeve having a gluing flap at one edge to fit a rectangular block. The block is to fit snugly without bulging the sleeve. The flap is to fit inside the pattern.
1. Sketch the block and dimensions (2 3/4" x 3 7/8").
2. Sketch pattern for a sleeve and list dimensions of each panel.
3. Draw pattern, score, and cut out pattern from B flute.

Lesson 14

I. Review homework.

   A. Students glue sleeves at gluing flap. With flap inside adjacent edge.
      1. Teacher anticipates poor choices of allowances in dimensioning of sleeves.
         a. Sleeves will be trapezodial in cross section.
            i. Teacher checks to see if block fits in sleeves.

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$2\frac{3}{4} + \frac{1}{8}$
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flap: $\frac{1}{2} + \frac{1}{16}$

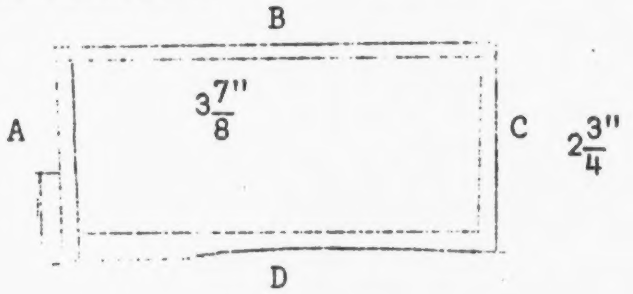
         b. Required dimensions of pattern from B flute.
            i. Teacher helps students analyze dimensions.
            ii. Students should check the dimensions on their sketches.
            iii. Note a space (on left side of sleeve above).
            iv. The sleeve has the shape of trapezoid.
      2. Students should make a second sleeve if the first was not satisfactory.
         a. Develop allowances for flaps with the class.
         b. Flap is to fit inside.

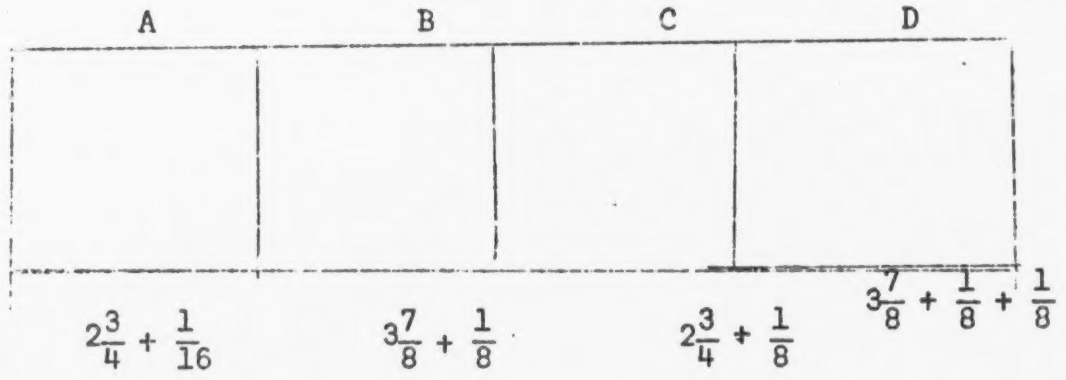
   B. Teacher introduces third type of sleeve design with flap fitting outside adjacent side.
      1. To avoid putting sleeve out of square allowance must be made on the panel to which the flap is attached.
         a. Cross section of sleeve.
            i. Allow for thickness of panel adjacent to the flap.

Lesson 14 (cont.)

Assignment: Complete the sketch for sleeve, draw, score, and cut out pattern for sleeve to fit block 2 3/4" x 3 7/8", having gluing flap outside the adjacent panel. Use a B flute.

Quiz Tomorrow.





A
B
C
D
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$3\frac{7}{8} + \frac{1}{8}$
$2\frac{3}{4} + \frac{1}{8}$
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Lesson 15

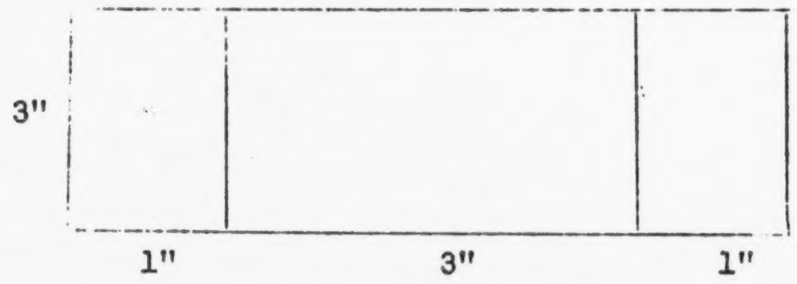
I. Quiz.

A. Sketch pattern of a sleeve to fit a block 1 1/2" by 4" and indicate allowances for a flap which is to fit inside. Use B flute.

II. Repeat study of allowances for folding corrugated using C flute.

A. Teacher demonstrates patterns made from C flute.
    1. Thickness is 3/32".

B. Each student receives a rectangular piece of C flute with dimensions 3" x 5".
    1. Measure 1" in from each short edge.
    2. Mark, score, and fold.



    3. Measure each panel where folds were made. Hold panels perpendicular.
    4. Note loss of dimension is a little less then 1/16" on each panel at each fold. (This can be only approximate).
    5. Note gain of measurement on outside of each panel is a little less than 1/16" at each fold.

C. Students should recognize same generalization:
    1. Half of the thickness of the corrugated is lost on each inside panel at each fold.
    2. Half the thickness is gained on the outside of the panel at each fold.

D. Classwork: Each student sketches pattern, draws, scores, and cuts pattern for sleeve made from C flute to fit a 2" x 4" block. (2 3/4" x 3 3/4"). Make pattern with ends which butt join on one face.

Assignment: Draw, score, and cut out two more patterns to fit the same block.
    1. With flap fitted inside adjacent panel.
    2. With flap fitted outside adjacent panel.

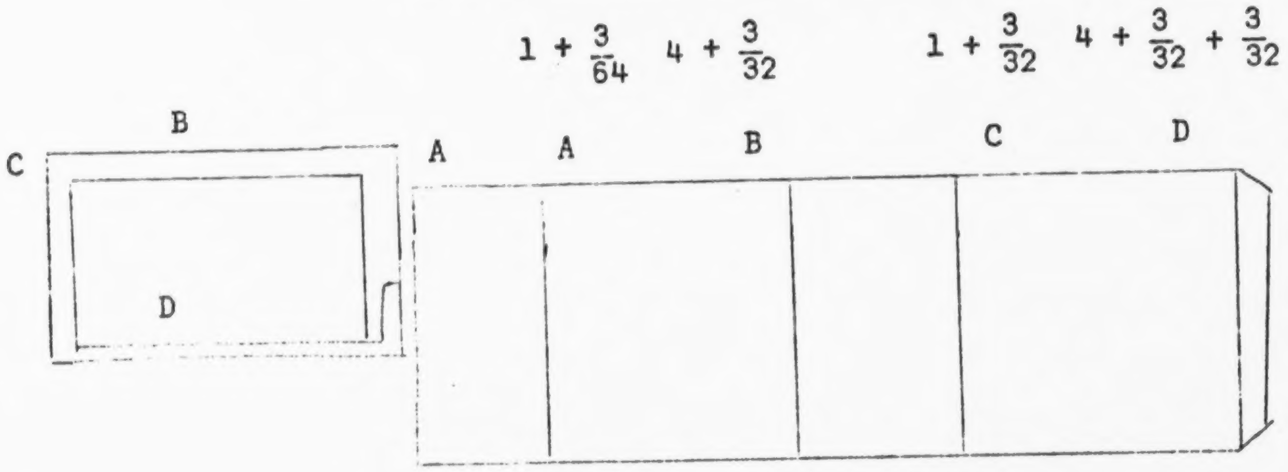
Lesson 16

I. Review homework.

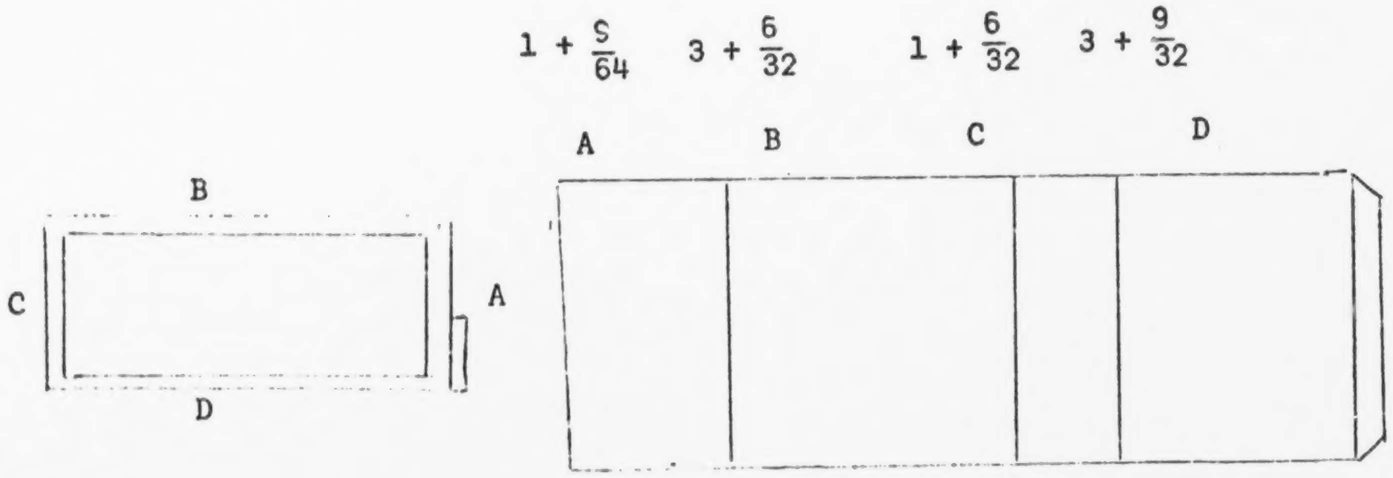
A. Teacher tests a few sleeves to determine fit.
   1. Notes where panels are short or long.
   2. Helps students analyze dimensioning of panels.

II. Classwork: Students design two sleeves from C flute so that inner sleeve will fit a block of wood 4" x 1" x 3".

A. Second sleeve fits at right angle to first so that block is covered on every end.
   1. Refer to the book mailing package of past lesson.
   2. Students must make allowances for folding of C flute.
      a. Pattern for the inner sleeve with the flap glued to outside of adjacent panel.



      b. Pattern for flap glued to outside outer sleeve.



Lesson 16 (cont.)

Assignment: Complete two sleeves, score, and cut out.
Calculate the area of the outer surface of the inner sleeve and the outer surface of the outer sleeve.

Lesson 17

I. Quiz.
   A. Sketch a pattern for a sleeve and indicate the allowances to fit a block 2" x 3" with a flap fitting inside the adjacent panel.
      The material to be used is E flute - 1/16" thick.
      1. Sketch on regular paper.

II. Review quiz and homework.
   A. Students glue flaps, teacher tests sleeves by attempting to slide block into each.
      1. Students compare dimensions. Sketch made on board.
      2. Calculations for area on the board.

III. Analysis of designs for patterns from corrugated board.
   A. Each student receives a copy of a commercially-made pattern from C flute (3/32" thick.)
      1. Reverse tuck design.
      2. Each student is to measure all outside dimensions of every panel of the pattern.
         a. Write each dimension on the pattern as one would on a plan.
         b. Also measure the gaps in the pattern and indicate these. (Gaps for locks on the dust flaps.)
      3. Next students write below each of the (above) measures the corresponding dimension when the box is to be folded.
         a. Compare results with their predictions.

Assignment: Complete dimensioning of the panels of pattern and your predictions for dimensions when the pattern is folded.

Repeat the classwork procedure in determing dimensions of each panel of the box if the box were to be made of A flute (3/16").

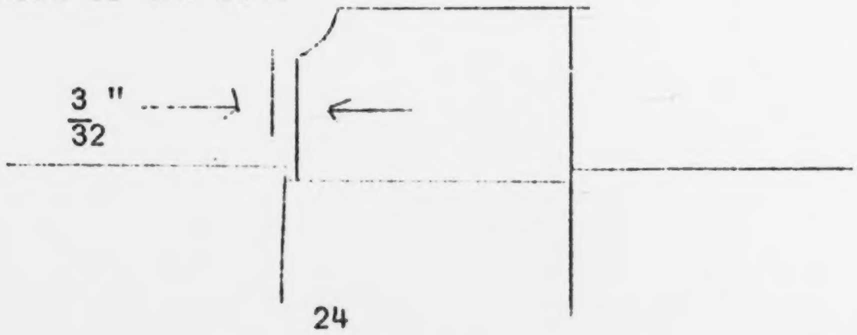
Lesson 18

I. Further analysis of corrugated box patterns.

A. Review homework.
1. Students predict inside dimensions of panels.
2. Students predict inside dimensions of the box.
3. Students fabricate the box and trace the outline of all overlapping parts onto the panels which they touch.
4. Open pattern and examine the pattern.
    a. Analyze the effect of two or three layers of corrugated board have in offsetting the outside measurements of the box.

B. Observations and analysis of the pattern.
1. Note: on C flute 3/64" is lost by each panel at each fold.
2. Note the direction of the fluting to give greatest strength to the box for stacking.
3. Note the way flaps are made to create locks on the tuck-top box.
    a. Note how loss in dimension caused by folding a dust flap caused the gap to be reduced in width, creating a lock.
        i. Example: Gap of 3/32" on the pattern will be diminished to 3/64" when flap is folded over. This causes a tight fit for the tuck top and creates lock. (See diagram below.)
4. Note offset in scoring of edges of side panels so that flaps will fold under top and top will fold flush.
    a. Estimate amount of offset in these scores.
        i. Use previously learned knowledge in gains and losses of corrugated according to thickness.

C. Classwork and assignment: Each student receives a second box pattern made by a commercial manufacturer. Use C flute.
1. Reverse tuck with a slit lock bottom.
2. Measure all edges of each panel. Write the dimensions on the panel as you would on a plan.
3. Measure the gaps which provide for locks.
4. Determine the inside dimensions of each panel and those of the box.



Lesson 19

I. Review homework.

   A. Students discuss various aspects of the pattern given for homework as discussed in Lesson 18.
      1. Be especially careful of allowances for the panels.
         a. Note allowances for offset of fold lines to provide for neat folds of one panel upon another
      2. Observe allowances for gaps for locks.
         a. Note loss of 3/64" in gap when dustflap is folded. at right angles for C flute.
      3. Students preduct inside and outside dimensions of the box.
      4. Students fabricate box and trace outline of all overlapping parts onto adjacent parts.
         a. Students measure inside and outside dimensions and compare with predicted measures.

II. Teacher challenges students to design and fabricate a Tuck Top Slit Lock Bottom carton to fit inside the carton used for homework.

   A. The material must be C flute.

   B. Students make sketch of the pattern, including dimensions.

Assignment: Complete design, cutting and scoring of TTSLB box to next in original box used in homework.
   1. Material must be C flute.
   2. Find the dimensions of the smallest rectangle from which the pattern can be cut.

Lesson 20

I. Quiz

A. Write in the required dimensions of a pattern for a Reverse Tuck design made from B flute. The inside dimensions of the box are indicated on an isometric view. The gluing flap is to fit on the inside of the adjacent panel.

1. Drawing on hectrograph.

II. Review quiz and homework.

A. Students glue flaps and test their pattern for fit into the commercially-made carton.
1. The teacher helps them check their design and construction.

B. Some students will be required to make a second attempt.
1. Students should compare their patterns with others by matching.

Assignment: Those students whose design was not satisfactory should make a second attempt.

Extra credit: Those whose designs were satisfactory should design two boxes which fit, side-by-side in the commercially-made box.

Use folding box board instead of corrugated board.

Determine the number of these patterns which can be cut from a rectangular sheet 18" x 20".

Lesson 21

I. Review homework.

   A. For extra-credit work: Students demonstrate the design and fabrication of the boxes to nest inside the large one.
      1. Sketches and dimensions on the black board.
      2. Number of patterns cut from a sheet 18" x 20".

II. Review of allowances required in proper dimensioning for folding corrugated board.

   A. Each student receives an undimensioned drawing of a Bumper End Book Folder.
      1. Students predict method of folding, purpose of the design.
      2. Obtain such a box pattern for demonstration.
      3. If the pattern is made from B flute, determine the inside dimensions.
         a. Teacher assists in the process of analyzing the dimensions.
      4. Note the type of lock, called a Mailing Lock.
      5. Note protective features of the design.
         a. All corners of the contents (a book) are protected.

Assignment: Teacher assigns two more problems. Give inside dimensions for a Bumper End Book Folder.
   1. Students are to determine the dimensions of the pattern:
      a. made from B flute; inside dimensions: 2" x 6" x 5".
      b. made from B flute: inside dimensions: 2 1/2" x 8" x 6".
   2. Each problem is to be presented on a different copy of the drawing for the pattern.

Lesson 22

I. Review homework.

  A. Sketches and dimensions on board.
    1. Teacher helps students analyze an actual pattern to determine the dimensions of the pattern to give the desired inside measurements.

II. Generalizations in dimensioning patterns for design in corrugated.

  A. Each student receives an unmarked drawing of a Bumper End Book Folder.
    1. Based upon the last assignment, dimension each panel for general dimensions l, w, and d of the contents.
      a. Note which dimensions are not dependent upon l, w, d.
    2. Represent the outer dimensions of the smallest rectangle from which the pattern can be cut.
      a. Note that these should be developed in general terms.

  B. Each student receives an unmarked drawing of a Reverse Tuck design
    1. Based upon the thickness of the fluting used (B flute) and the general dimensions l, w, and d, of the contents, represent the dimensions of each panel.
      a. Note which dimensions are not dependent upon l, w, or d.
    2. Represent the outer dimensions of the smallest rectangle from which the pattern can be cut.
      a. Note that these should be developed in general terms.

  C. Classwork and assignment.
    1. Dimension all patterns given in these drawings based upon the general dimensions l, w, and d, of the contents. Assume the material is B flute.
      a. Students receive an unmarked drawing of a Regular Slotted Carton
      b. Students receive an unmarked drawing of a Tuck Top Slit Lock Bottom Box
    2. For each pattern represent in general terms the dimensions of the smallest rectangle from which each pattern can be cut.

Lesson 23

I. Representing algebrically the overall dimensions of a pattern.

   A. Review homework.
      1. Sketches and dimensions on the board.
      2. Test the overall dimensions represented algebrically by comparison with the actual patterns used in earlier lessons.
         a. Substitute measures from pattern and evaluate algebraic expressions for dimensions of pattern.
         b. Measure overall dimensions of pattern and compare results with (a).
         c. Add measures of panels to obtain overall dimensions and comprre results with (a) ndd (b).
      3. Repeat this process with other box patterns.

Assignment: Use the drawings of patterns used in Lesson 22 to represent the overall dimensions of the pattern using algebraic variables for l, w, and d.

Follow the same steps used in class to test that your final expressions for overall dimensions are correct.

Lesson 24

I. Quiz.

- A. Each student receives an unmarked drawing of a Bumper End Book Folder.
    - 1. Determine the dimensions indicated based upon the inside dimensions of the box and the use of B flute.

II. Review quiz and homework.

- A. Algebraic expressions on board and calculations of overall dimensions for comparison.

III. Introduce plans for cooperative projects for the next ten lessons.

- A. Part of each period will be used for the study of introductory algebra based upon the concepts introduced in the last two lessons.
    - 1. Homework assignments will be made on hectographed pages. Reference will be made to the textbook.

- B. The remainder of each period will be devoted to cooperative planning by small groups of students.
    - 1. Each group will be expected to complete at least two projects.
        - a. Plan must be first approved by the teacher.
        - b. Sketches in isometric and sketches of patterns must be made by each student.
        - c. Students must agree upon the type of paper board to use.
    - 2. Each group must submit the first project, completed, within one week, including:
        - a. Drawings carefully done, with dimensions.
        - b. Objects fabricated and fitted to specifications.
        - c. Pages of calculations, complete with explanations.
        - d. Above set up ready for display.

IV. Teacher describes possible projects for the first week.

- A. Carton made of three pieces, one for each student.
    - 1. Bottom made of B flute, with open top.
    - 2. Top, made of B flute, open at the bottom.
    - 3. Sleeve, made of C flute, fitting full height of the box.
        - a. Sleeve fits tightly inside other parts.

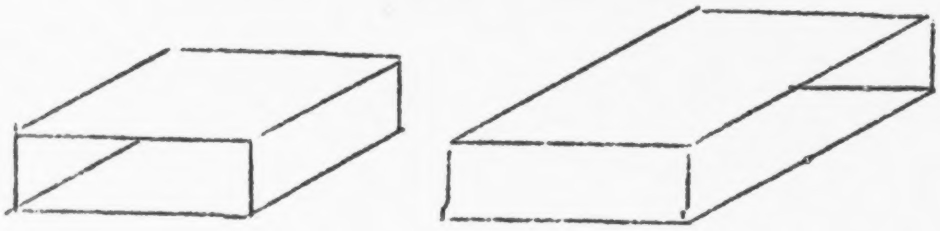
- B. Large carton which folds two smaller cartons.

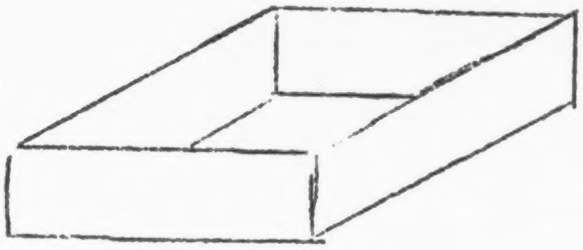
Lesson 24 (cont.)

1. For a group of three students.
   a. Large carton of B flute.
   b. Smaller boxes of C flute. These cannot be the same size. The two smaller boxes nest inside the larger box.

C. Similar to part (B) with the use of trapezoidal patterns for paper boxes which nest.

D. Develop three inter-lining sleeves to protect two thick books. (May use our large Shop Mathematics textbook.)
   1. Inner lining wraps around side edges and binding of the books. Made of B flute.
   2. Middle lining wraps around bottom and top edge of books. Made of C flute.
   3. Outer lining wraps around edges and binding of book and encloses the first two liners.
   4. Each lining has a gluing flap.





Lesson 25.

I. Further experience with addition of algebraic expressions.

  A. Each student is given a hectographed page of a different pattern for a paper box.
    1. Students analyze the pattern to determine how it is fabricated.
    2. Students then attempt to fabricate an actual copy of the pattern.
    3. Students represent the dimensions of each panel of the pattern algebraically in terms of l, w, and d of the inside dimensions of the box.
    4. Students represent the overall dimensions of the box in terms of l, w, and d.
      a. Note which panels are not dependent upon l, w, or d.

II. Continue plans for cooperative project.

Assignment: Complete part I (above).

Lessons 26 through 29. Follow similar plan to Lesson 25.

Lesson 30.

I. Quiz on areas and allowances for folding of patterns for paper boxes.